# Lecture 7 (part II)

## Testing dense graphs

- bipartiteness

# Adjacency Matrix model

$G$ represented by matrix $A$
s.t. can query $A$ in
one step

$$A = \begin{bmatrix} & & \\ & A_{ij} & \\ & & \end{bmatrix}$$

1 if $(i,j) \in E$
0 o.w.

## Distance from property P:

<u>def</u> $G$ is $\varepsilon$-far from $P$ if must change $> \varepsilon \cdot n^2$
entries in $A$ to turn $G$ into member of $P$

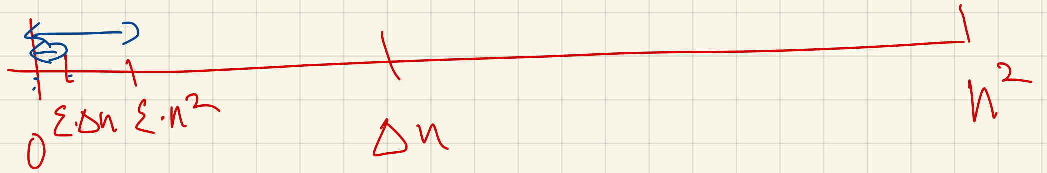
## Testing "sparse" properties:

<u>all</u> graphs are $\varepsilon$-close to connected in this model

$\Rightarrow$ trivial tester outputs "PASS" w/o looking at graph

| | Graph type | max degree | natural representation | notion of distance | |
|---|---|---|---|---|---|
| Previously | sparse | $\Delta$ | adjacency list | $\le \varepsilon \cdot \Delta \cdot n$ | edges changed |
| Now | dense | $n$ | adjacency matrix | $\le \varepsilon \cdot n^2$ | " " |

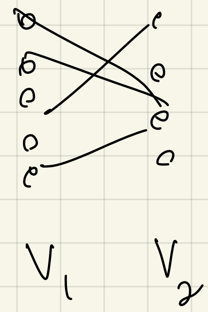
Should be easier to detect



# Bipartiteness:

- Can color nodes red/blue s.t. no edge monochromatic
- Can partition nodes into $(V_1, V_2)$ s.t.

$$\nexists \underset{(u,v)}{\overset{}{e}} \in E \quad \text{s.t.} \quad \begin{array}{l} u,v \in V_1 \\ \text{or } u,v \in V_2 \end{array}$$




equivalent definitions

not bipartite $\Rightarrow$ $\forall (V_1, V_2)$ $\exists$ "violating edge"

# $\varepsilon$-far from bipartite: (definition)

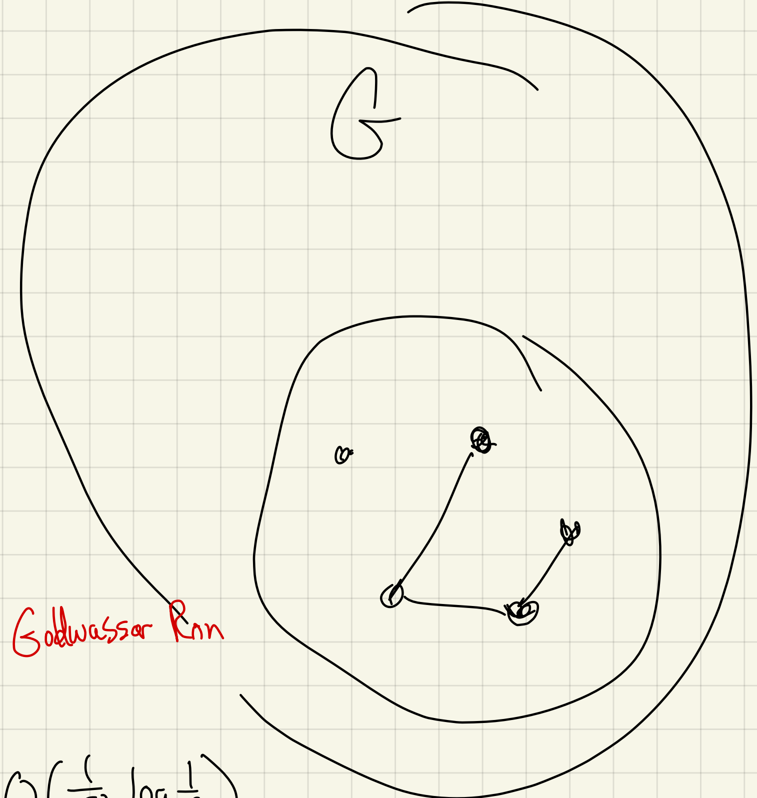
- must remove $> \varepsilon \cdot n^2$ edges to make bipartite
- $\forall$ partitions $V = (V_1, V_2)$, $> \varepsilon \cdot n^2$ violating edges

equivalent

## Testing Algorithms:

- Testing exact bipartiteness:

  e.g. BFS



- Proposed testing algorithm:

Goldreich Goldwasser Ron

- Pick sample of nodes of size $O(\frac{1}{\varepsilon^2}\log\frac{1}{\varepsilon})$
- Consider induced graph on sample

  ignore nodes not in sample
  ignore edge s.t. $\geq 1$ endpt is not in sample

- If bipartite, output PASS
  else output FAIL

  e.g. BFS

This actually works!!

# A first attempt at a proof?

if $G$ bipartite, induced graph is bipartite, so algorithm passes

if $G$ $\varepsilon$-far from bipartite:

must remove $\varepsilon n^2$ edges to make it bipartite

equivalently:

$\forall$ partition $V_1, V_2$ have $> \varepsilon n^2$ violating edges ($> \varepsilon$ fraction of slots in adj matrix)

$\Rightarrow$ $\forall (V_1, V_2)$ a sample of edges of size $\geq \Theta\left(\frac{1}{\varepsilon} \log \frac{1}{\delta}\right)$

hits a $(V_1, V_2)$-violating edge with prob $\geq 1 - (1-\varepsilon)^{\frac{c}{\varepsilon} \log \frac{1}{\delta}}$

$\geq 1 - e^{-c \cdot \log \frac{1}{\delta}} = 1 - e^{-\log 1/\delta} = 1 - \delta$

(set $c = 1$)

Great! ?

need to hit violating edge for every partition

how is this an algorithm?

no edge violates all partitions