Lossy Encryption from General Assumptions

Brett Hemenway and Rafail Ostrovsky

Crypto in the Clouds Workshop, MIT

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Outline

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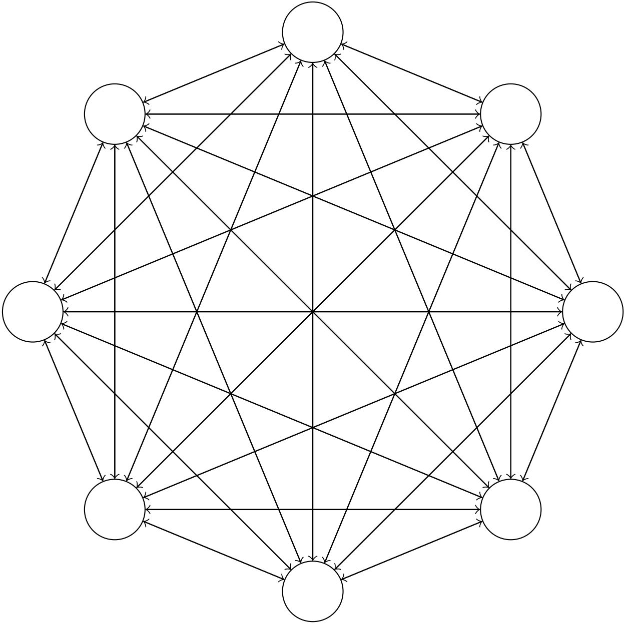

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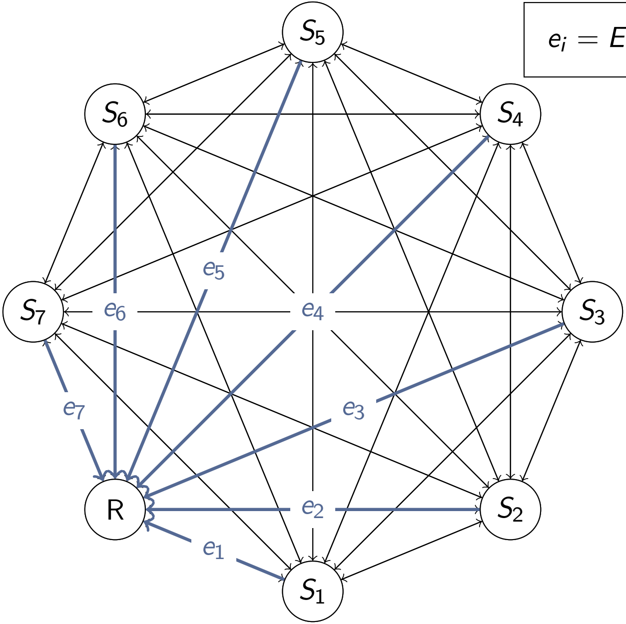

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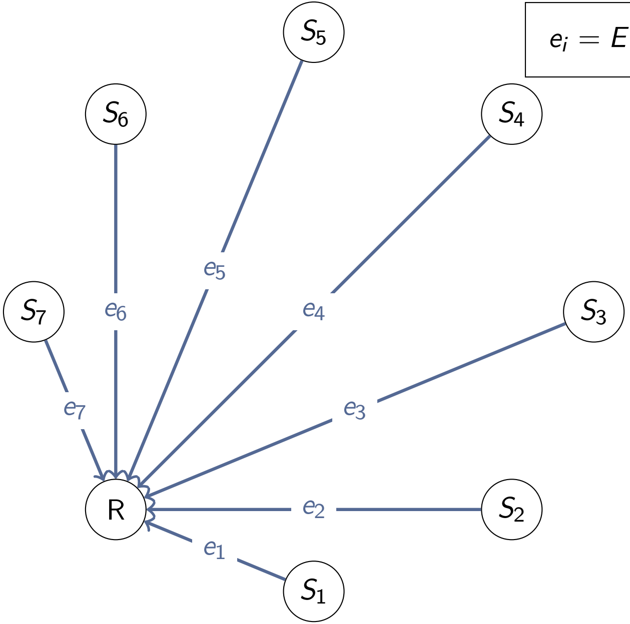

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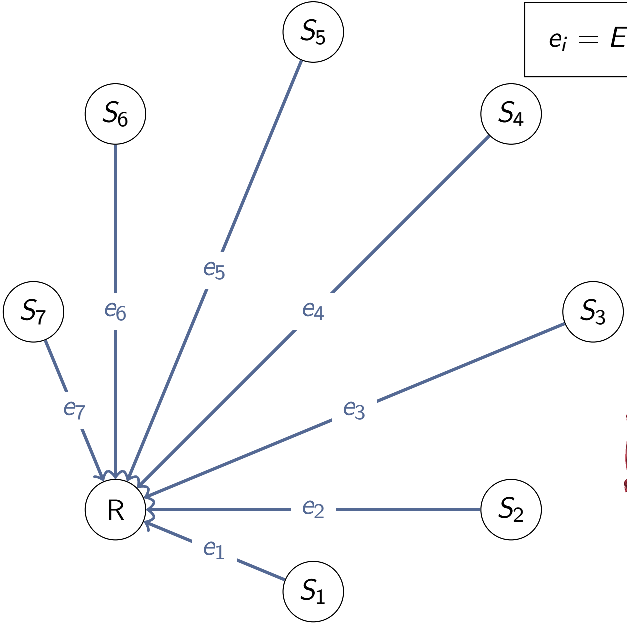

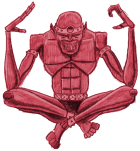

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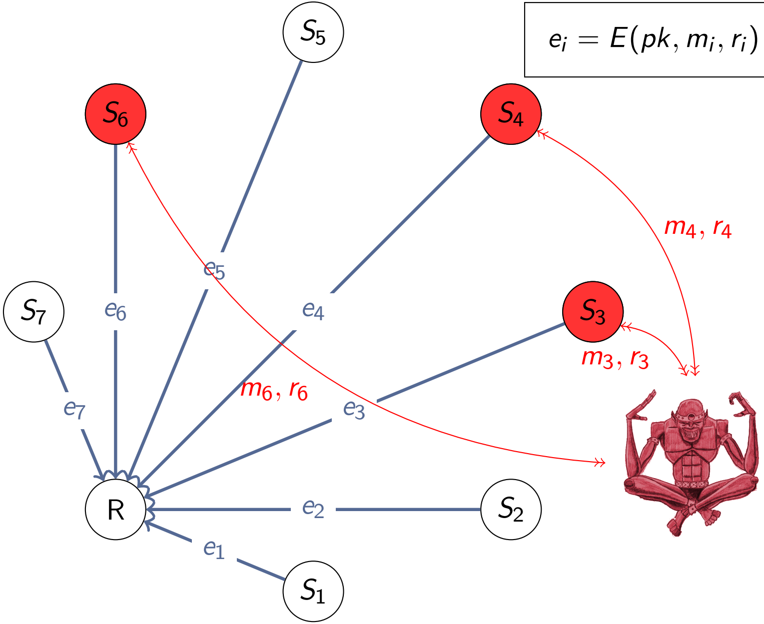

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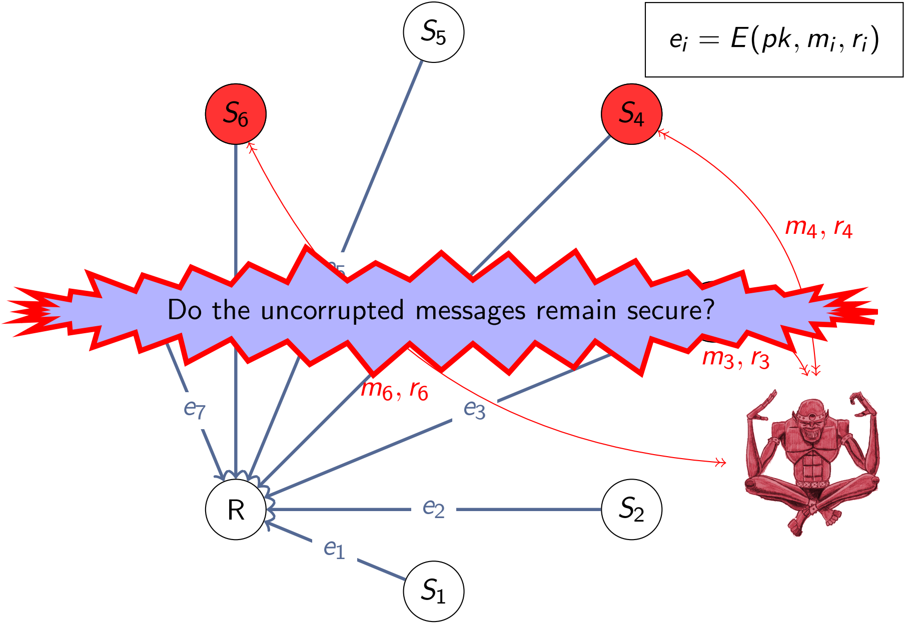
$e_i = E(pk, m_i, r_i)$
S_5
S_6
S_4
m_4, r_4
Do the uncorrupted messages remain secure?
m_3, r_3
m_6, r_6
e_7
e_3
R
e_2
S_2
e_1
S_1

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- No one has been able to exhibit an IND-CPA secure system that is not IND-SOA security.

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$$\left| \Pr\left[A^{IND-SO-ENC-REAL} = 1\right] - \Pr\left[A^{IND-SO-ENC-IDEAL} = 1\right] \right| < \nu$$

Lossy Encryption in Detail

$$G(1^\lambda, mode), E(pk, m, r), D(sk, c)$$

Correctness:

For all m, r

$$D(E(pk_I, m, r)) = m$$

Lossiness:

For all m_0, m_1

$$\{E(pk_L, m_0, r)\} \approx^s \{E(pk_L, m_1, r)\}$$

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Notice: Indistinguishability + Lossiness $\Longrightarrow$ IND-CPA security

Lossy Encryption is IND-SO-ENC Secure (BHY09)

In Lossy mode, the distributions

$$(E(m_1, r_1), \ldots, E(m_n, r_n)) \approx^s (E(m'_1, r_1), \ldots, E(m'_n, r_n))$$

Since the encryptions are statistically independent of the messages, so even after conditioning on certain openings, the rest remain independent of the messages.

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  - Statistical rerandomization:

$$\{\text{ReRand}(E(pk, m, r))\} \approx^s \{\text{ReRand}(E(pk, m, r'))\}$$

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If $E(pk, m, r)E(pk, m', r') = E(pk, m + m', r^*)$, then we can re-randomize by doing

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If you can sample statistically close to uniformly from the set of encryptions of 0 then homomorphic encryption is statistically rerandomizable

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The indistinguishability of modes follows immediately from the Semantic Security of (G, E, D).

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- We can make lossy encryption, simply by setting $PK = (pk, e)$ where $e = E(pk, 0, r)$ in Lossy Mode and $E(pk, 1, r)$ in injective mode.

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- Decryption is the same.

Oblivious Transfer Implies Lossy Encryption

Sender

Receiver

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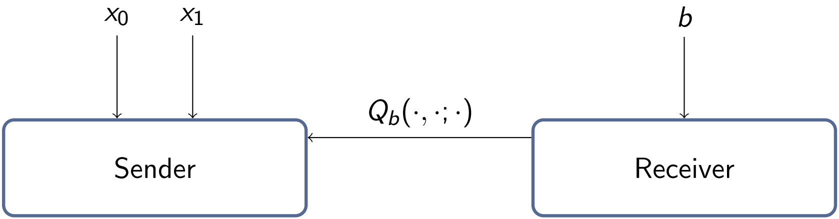

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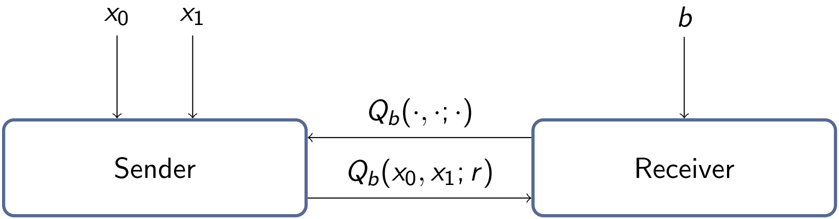

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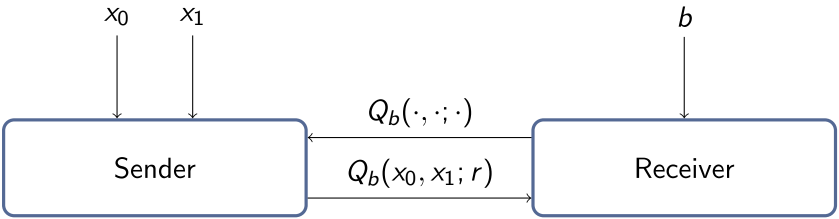

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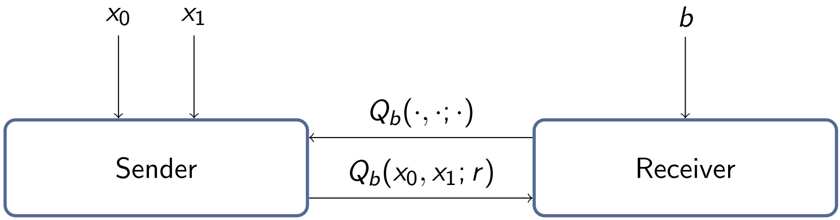

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Computational receiver privacy implies indistinguishability of modes

Statistical sender privacy implies lossiness of lossy branch

Chosen Ciphertext Security in the Selective Opening Setting

IND-SO-CCA2: Definitions

Challenger

Adversary

IND-SO-CCA2: Definitions

Challenger

Adversary

Decryption Queries

IND-SO-CCA2: Definitions

Challenger

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Decryption Queries

Selective Opening Query

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Output b

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Challenger | Adversary

$$\xleftarrow{\quad c \quad}$$

$$\xrightarrow{\quad D(c) \quad}$$

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Selective Opening Query

Decryption Queries

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c

$D(c)$

$\vdots$

$E(m_1, r_1), \ldots, E(m_n, r_n)$

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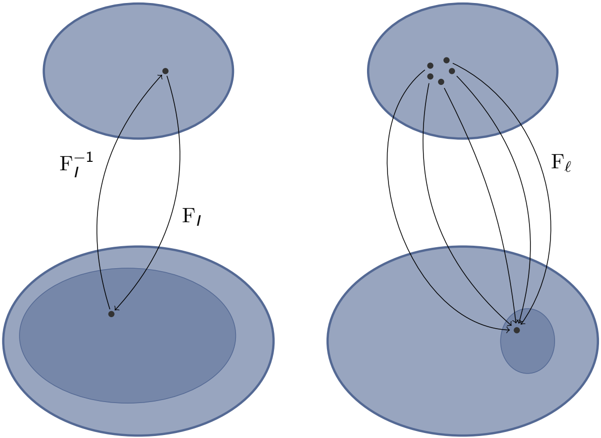
Lossy Trapdoor Functions [PW08]

$$\mathrm{F}_I \approx \mathrm{F}_\ell$$



Injective Mode

Lossy Mode

Lossy Trapdoor Functions in Detail

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All-But-One Functions [PW08]

$$(s, t) \leftarrow G_{ABO}(1^\lambda, b^*)$$

Trapdoor:

For $b \neq b^*$

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Can be constructed from LTDFs

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IND-SO-CCA Construction

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$$\text{sig} = \text{Sign}_{sk}(F_{LTDF}(s_0, x), F_{ABN}(s_1, vk, x), h(x) \oplus m),$$

output the ciphertext:
$(vk, F_{LTDF}(s_0, x), F_{ABN}(s_1, vk, x), h(x) \oplus m, \text{sig})$

A SEM-SO-CCA Secure Construction

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  - The statistically hiding property of lossy encryption allows us to prove IND-SO security.
    Statistical NIZKs should allow us to prove IND-SO-CCA security.

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- **Honest-Prover State Reconstruction:** There exists a simulator that can create a proof P without a witness, then, given a witness w can produce randomness r such that P appears to have been generated with w and r.

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$$\overline{\pi} = (\pi_1, \ldots, \pi_\ell) = (\mathrm{Prover}(\sigma_i, (e_0, e_1), w), r_i^{nizk})_{i \in \mathfrak{g}(vk)}$$

$$\mathrm{sig} = \mathrm{Sign}(e_0, e_1, \overline{\pi}),$$

output the ciphertext: $c = (vk, e_0, e_1, \overline{\pi}, \mathrm{sig})$.

Theorem

This construction is SEM-SO-CCA2 Secure

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- What about receiver corruption?

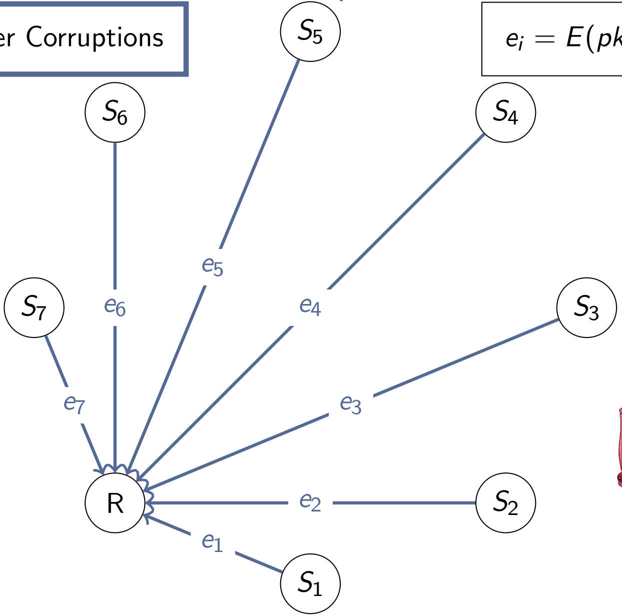
Open Question: Receiver Corruption

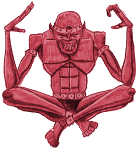
Recall: Sender Corruption Game

Open Question: Receiver Corruption

Sender Corruptions

$$e_i = E(pk, m_i, r_i)$$





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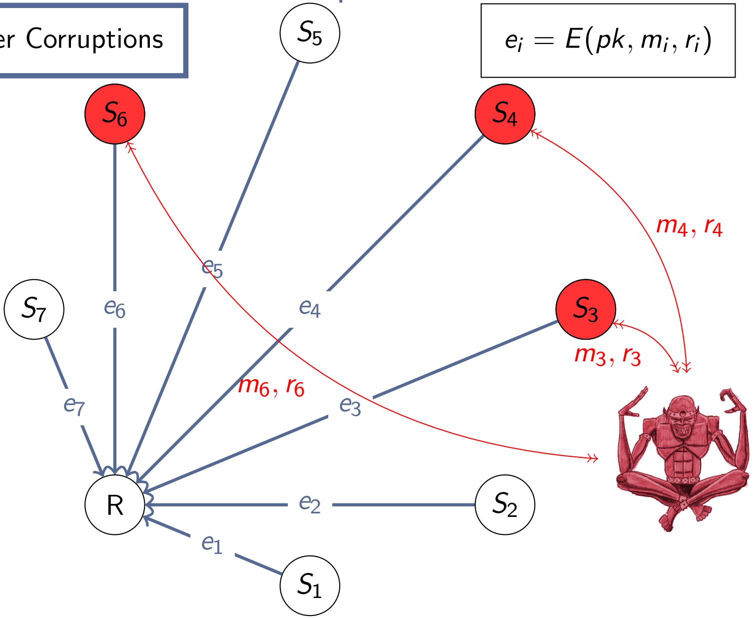

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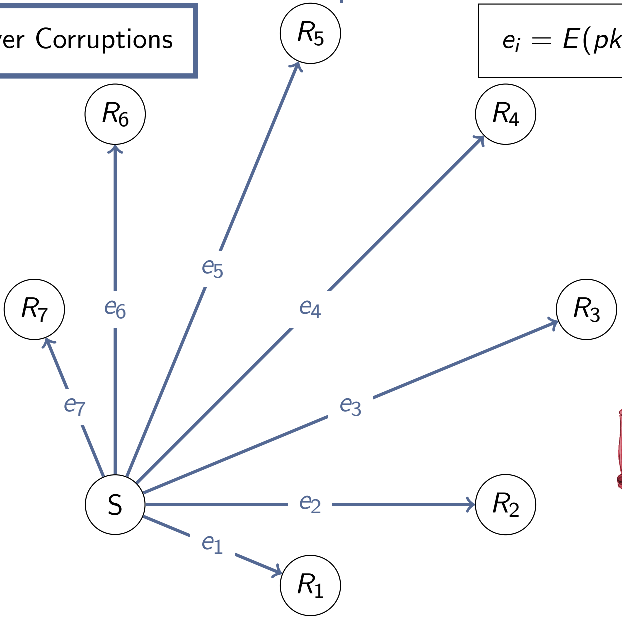

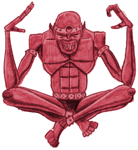

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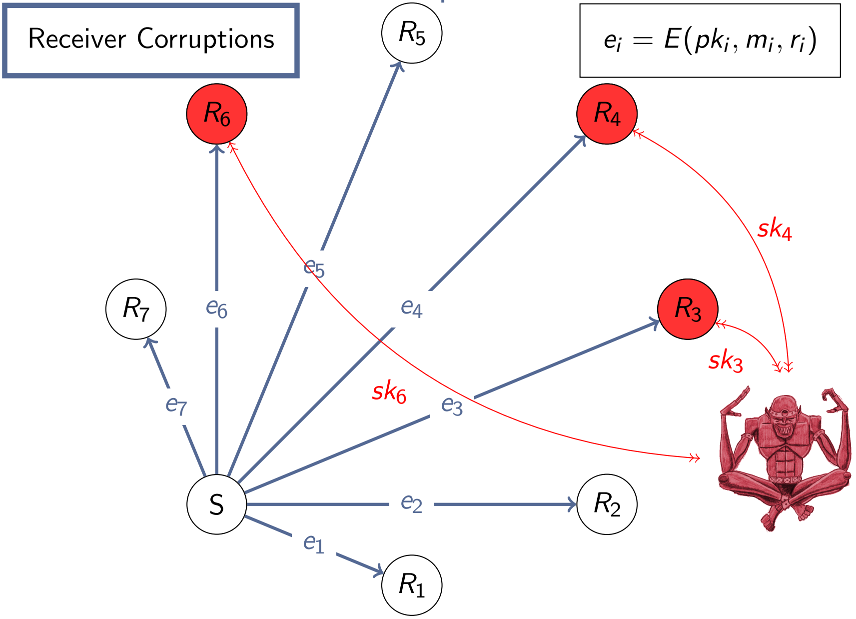

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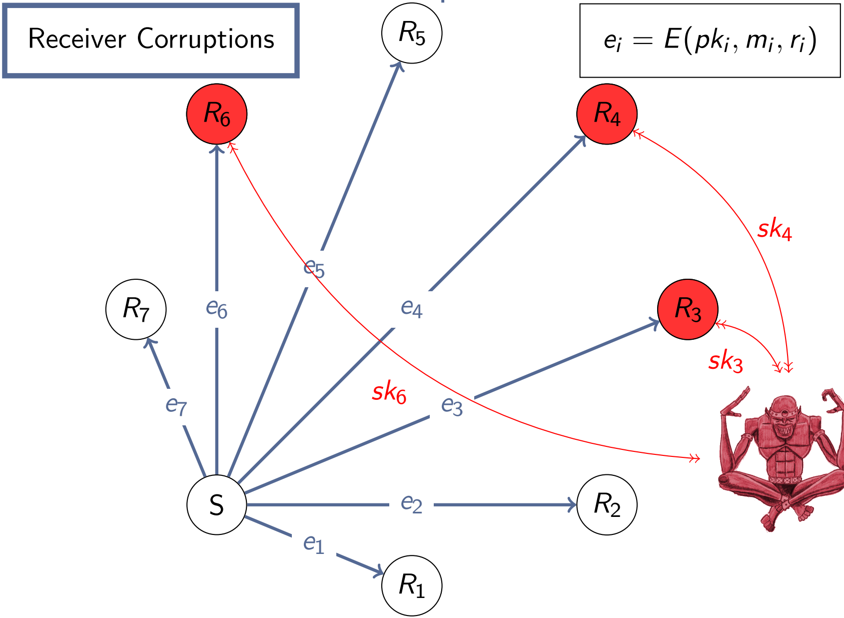

Thanks!