

Adventures in Maze Folding Art

ERIK D. DEMAINE^{1,a)} MARTIN L. DEMAINE^{1,b)}

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Abstract: Every orthogonal graph, extruded orthogonally from a rectangle, can be folded from a rectangle of paper a constant factor larger. This computational origami result was proved a decade ago, and has since enabled the design of a mathematical/puzzle font and a variety of art prints. Here we survey the maze-folding art prints we have designed.

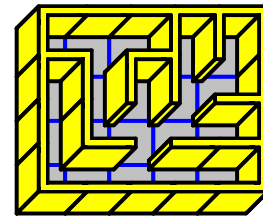
1. Maze Folding

Consider an orthogonal 3D “maze”, formed by extruding some of the integer grid edges of an $x \times y$ rectangle orthogonally to a constant height h , such as the maze in Fig. 1a. In 2010, we proved with Jason Ku [2] that every such 3D maze can be folded from a piece of paper of dimensions $(2h + 1)x \times (2h + 1)y$, that is, just a factor of $2h + 1$ larger than the base rectangle. In particular, when the extrusion height $h = 1$, the scale factor is just 3, independent of the complexity of the maze. The algorithm for computing the resulting crease pattern is implemented in a freely available web app,^{*1} which generated the example in Fig. 1. Fig. 1b shows a real-life example of folding such a maze. More recently, joint work with Jason Ku and Madonna Yoder [4] proved similarly efficient results for triangular and hexagonal “mazes”.

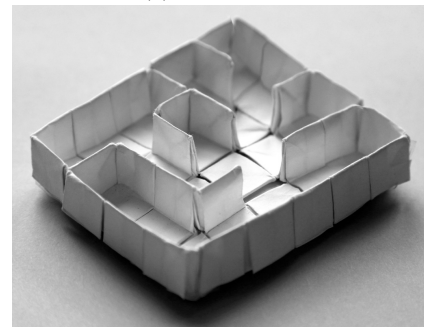
2. Maze Font

Also in 2010, we developed with Jason Ku [3] an orthogonal maze *font*: an orthogonal grid graph that looks like each English letter and numeral. By the result above, any collection of such letters/numerals can be efficiently folded from a rectangle of paper. This font was one of the first in a long series of mathematical fonts that we have designed, each based on a mathematical theorem or open problem. For more, see [1] and our website.^{*2} Like many of these fonts, the maze folding font offers a *puzzle font*: displaying the crease pattern alone encodes a hidden message that is a puzzle to decipher, either by folding or by careful analysis.

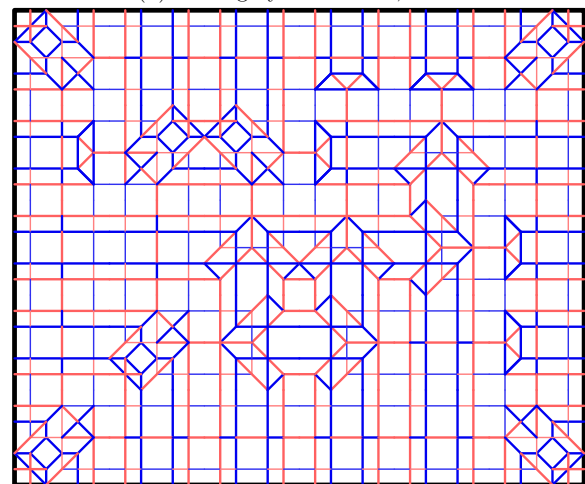
Fig. 2 shows a new print illustrating the maze folding font. It shows an imagined folding of the entire alphabet from a single rectangle of paper. Notably, it shows the correspon-



(a) Input maze.



(b) Folding by Chris Chin, 2010.



(c) Crease pattern.

Fig. 1: Example of maze folding algorithm [2], from input (a) to crease pattern (c) to real-world folding (b). © Erik Demaine and Martin Demaine

¹ MIT Computer Science and Artificial Intelligence Laboratory, 32 Vassar St., Cambridge, MA 02139, USA

a) edemaine@mit.edu

b) mdemaine@mit.edu

*1 <http://erikdemaine.org/fonts/maze/>

*2 <http://erikdemaine.org/fonts/>

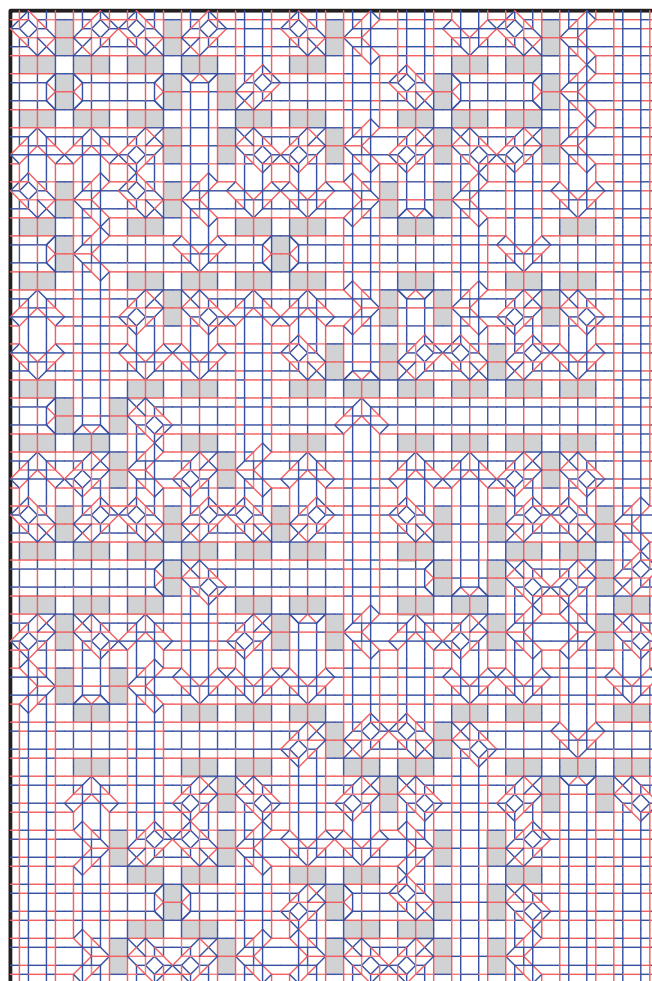
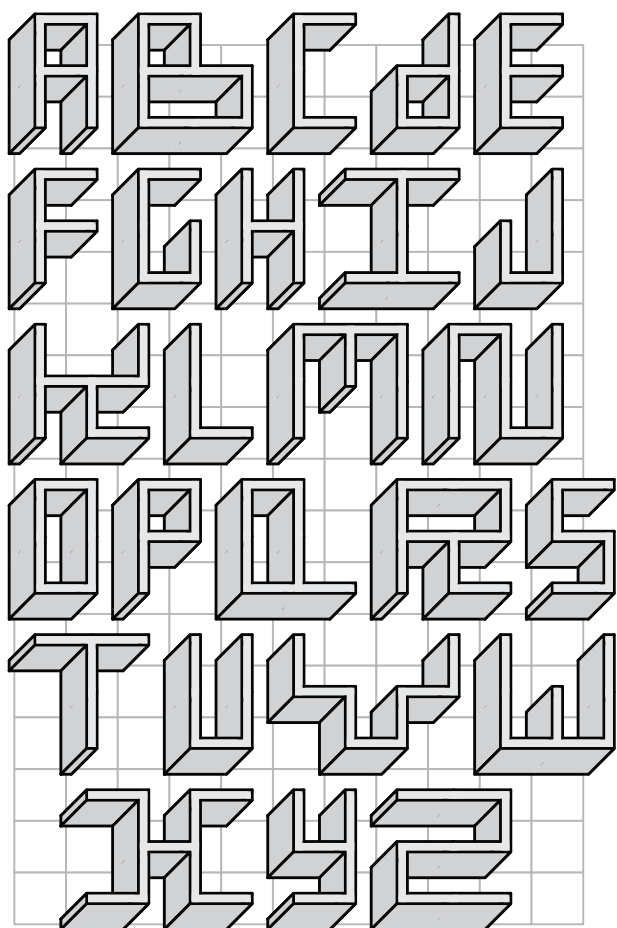


Fig. 2: “Maze Folding Alphabet” (2019), first exhibited at the National Museum of Mathematics. © Erik Demaine and Martin Demaine

dence between the crease pattern and the resulting letters by shading in gray the square faces that end up folding into the vertical extruded faces of the 3D letters. Even with this correspondence shown, it is tricky to recognize the letters in the crease pattern: mentally rotate each of the 1×2 gray rectangles by 90° to make the letters appear.

An interesting challenge is to extend our maze folding font to other languages. The orthogonal restriction considered here makes this difficult for many characters, but some are easier. Fig. 3 shows an example with two Japanese/Chinese characters, 三 (three) and 井 (well).

3. Shadow Text

Our first art print to incorporate the maze folding font is joint work with Sarah Stengel, an artist in New York. Fig. 4 shows this “Yes/No” print, originally designed in 2011 and slightly redesigned for this article. This print shows a crease pattern on the top and folded form on the bottom. In addition to creases, the top pattern includes shaded gray regions that do not look like much by themselves. When folded, the cryptic crease pattern forms the 3D structure of the word “YES” while the shaded regions come together to form the word “NO” as a kind of false shadow. This piece therefore

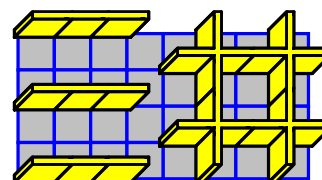
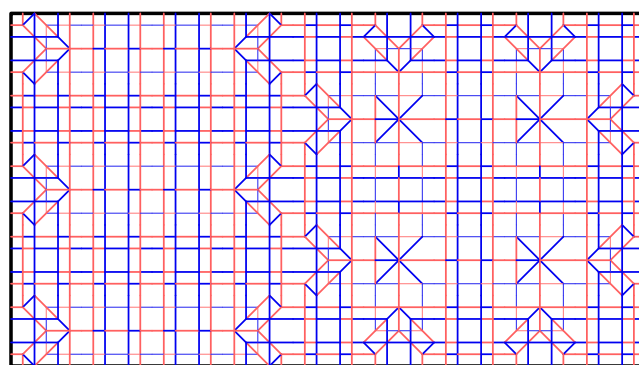


Fig. 3: Example made for a 2018 visit by Mitsui (三井). © Erik Demaine and Martin Demaine

represents the classic yes/no conflict, in a puzzle form on the top and a revealed form on the bottom.

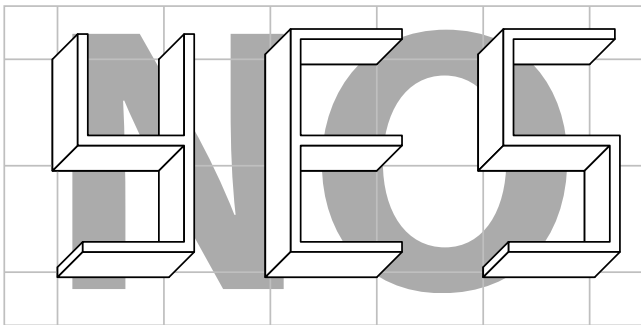
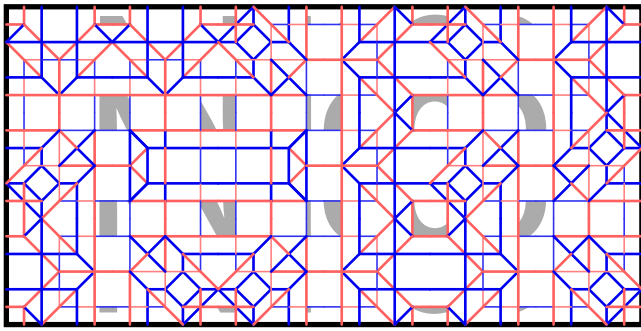


Fig. 4: “Yes/No” (2011). © Erik Demaine, Martin Demaine, and Sarah Stengel

This crease pattern is simple enough that it can be folded in real life. Fig. 5 shows the result.



Fig. 5: Folding of Fig. 4 by Eli Davis and Jennifer Ramseyer, 2012. © Erik Demaine and Martin Demaine

Returning to the challenge of other languages, we made a Korean version of “Yes/No” in 2016. There are a few forms of “yes” and “no” in Korean to choose from. The two standard forms of “yes” are 예 (“ye”) and 네 (“ne”); we chose the latter because it is orthogonal and primary since 1988. The polite form of “no” is 아니요 (“aniyo”); luckily, the background image does not need to be orthogonal. Fig. 6 shows the resulting print.

4. Multiple Views

In our MIT folding class (6.849: Geometric Folding Algorithms), a student George Xing came up with another way to represent the yes/no concept (inspired by our “Yes/No” print). Fig. 7 shows the maze he designed, which does not look like much in a generic 3D state. But looking down each

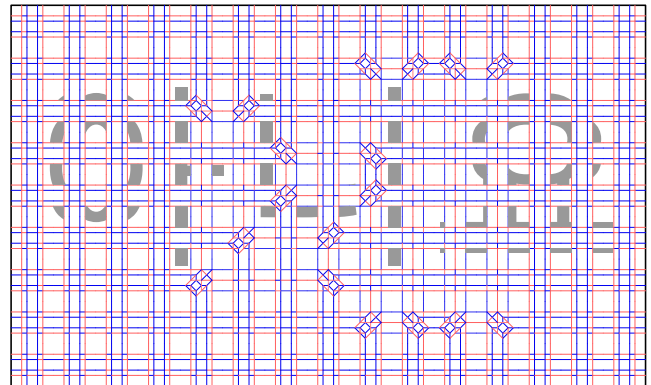
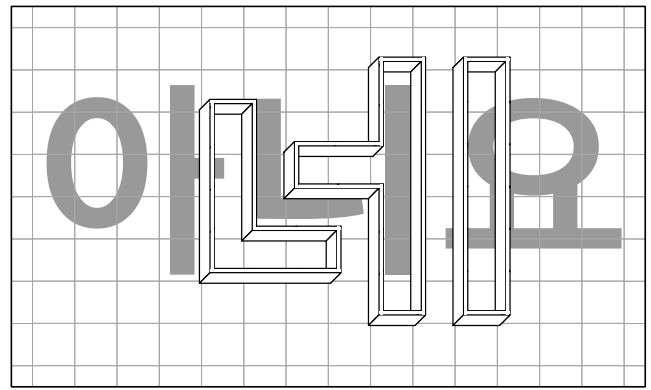


Fig. 6: “Korean Yes/No” (2016). © Erik Demaine and Martin Demaine

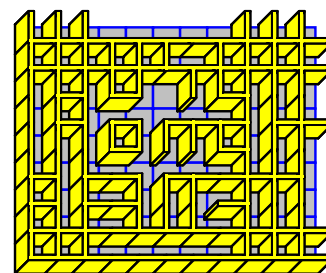
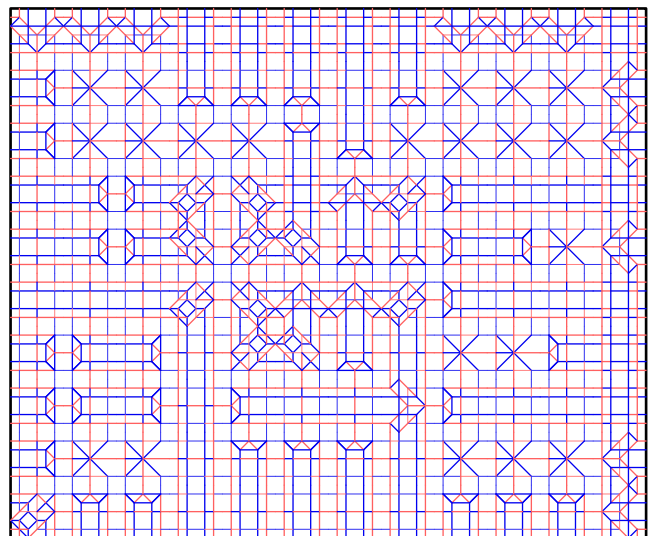


Fig. 7: What secret messages are hidden within this maze? See Fig. 8 for the answer. Design by George Xing (2012); drawing © Erik Demaine and Martin Demaine

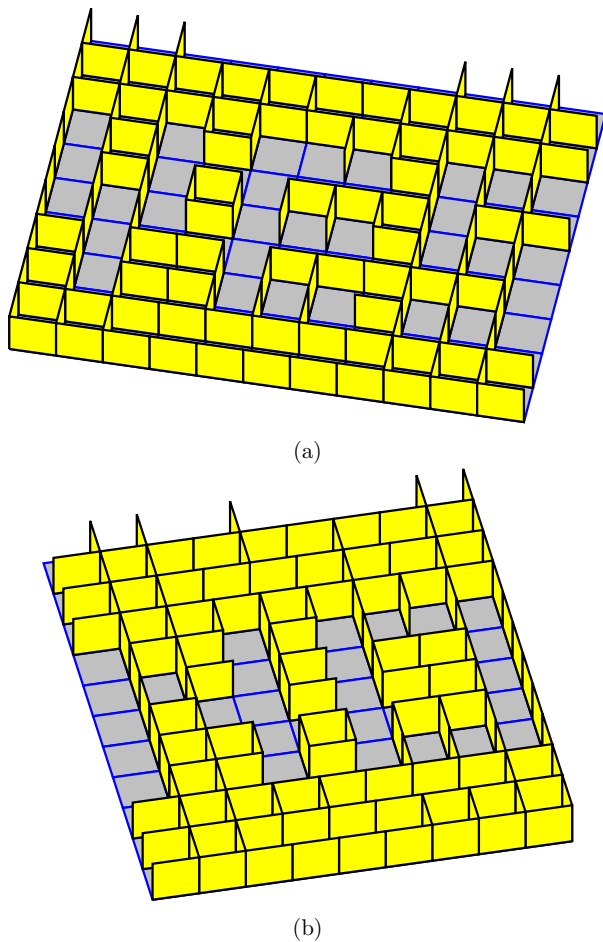


Fig. 8: Axis-aligned views of the maze from Fig. 7 reveal two secret messages. Design by George Xing (2012); drawing © Erik Demaine and Martin Demaine

axis of the maze (in an orthographic view) reveals two secret messages, shown in Fig. 8.

5. Shadow Image

The same shadow idea from “Yes/No” works as well for images as text. Fig. 9 shows an art print prepared for and presented during Erik’s plenary talk at JCDCCGG 2019. On the top is a crease pattern with small images that do not appear to be anything special. On the bottom is the result after folding, revealing hidden text in the 3D shape (Jin Akiyama’s name, in a common Japanese order), while the background coalesces to form a hidden photograph of Jin Akiyama.

6. Conclusion

Maze folding has provided a surprisingly rich playground for designing art prints. We encourage you to try designing your own, and send us what you come up with!

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Erik D. Demaine received a B.Sc. degree from Dalhousie University in 1995, and M.Math. and Ph.D. degrees from the University of Waterloo in 1996 and 2001, respectively. Since 2001, he has been a professor in computer science at the Massachusetts Institute of Technology. His research interests range throughout algorithms, from data structures for improving web searches to the geometry of understanding how proteins fold to the computational difficulty of playing games. In 2003, he received a MacArthur Fellowship as a “computational geometer tackling and solving difficult problems related to folding and bending— moving readily between the theoretical and the playful, with a keen eye to revealing the former in the latter”. He cowrote a book about the theory of folding, together with Joseph O’Rourke (*Geometric Folding Algorithms*, 2007), and a book about the computational complexity of games, together with Robert Hearn (*Games, Puzzles, and Computation*, 2009). With his father Martin, his interests span the connections between mathematics and art.

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Martin L. Demaine is an artist and computer scientist. He started the first private hot glass studio in Canada and has been called the father of Canadian glass. Since 2005, he has been the Angelika and Barton Weller Artist-in-Residence at the Massachusetts Institute of Technology.

Martin works together with his son Erik in paper, glass, and other material. Their artistic work includes curved origami sculptures in the permanent collections of the Museum of Modern Art in New York, and the Renwick Gallery in the Smithsonian. Their scientific work includes over 140 published joint papers, including several about combining mathematics and art.

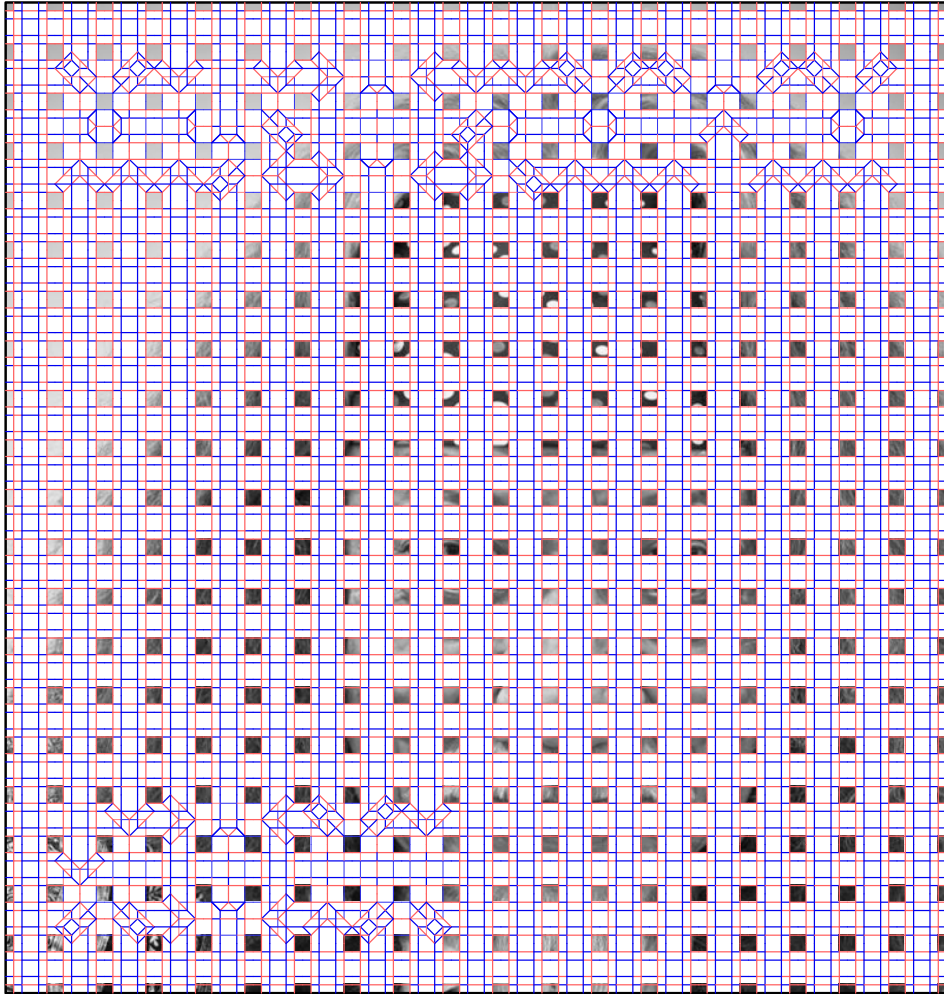


Fig. 9: "Oriyama Jin" (2019). Presented at JCDCGGG 2019 in honor of Jin Akiyama. © Erik Demaine and Martin Demaine