Abstract

Intractability results for optimization problems complement algorithm design techniques by proving what cannot be computed efficiently and therefore suggests the use of alternative simplifications of the problem in order to efficiently solve it. The rationale behind this approach is that problems are seldom encountered in their utmost generality in practice. Two particularly relevant approaches to tackle "hard" problems are: (i) Restrict the problem to the case of better structured inputs, and (ii) Allow sub-optimal (but provably good) *approximate* solutions. The purpose of this thesis is to prove intractability results even under these two simplifications.

Graphs are the most pervasive discrete structures used to model computational problems and our work focuses on graph–theoretic problems. The first part of our work proves **NP**-hardness results corresponding to situation (i) above, where the input graphs are restricted to belong to special classes like bipartite, planar, chordal, split, bounded–degree etc. More specifically, we consider two optimization problems: (a) Minimum Graph Spanners, and (b) Graph Packing Problems.

Regarding graph spanners, we prove that the problem of finding a t-spanner with fewest possible edges is **NP**-hard on chordal and bounded–degree graphs for $t \geq 2$, on bipartite graphs for $t \geq 3$, and on split graphs for $t = 2$. Regarding packing problems, we prove that the problem of finding a maximum number of pairwise vertex–disjoint triangles is **NP**-hard on chordal, planar, line and total graphs, but can be solved in polynomial time on split graphs. We also prove that the problems of ascertaining the existence of a P_3-factor in degree-3 bipartite graphs and of a K_4-factor in comparability graphs are **NP**-complete.

The second part of the work is concerned with (in)approximability of optimization problems, developments relating to which are arguably the most significant ones in complexity theory during this decade. We first provide an example of a family of optimization problems which behave better and better with respect to approximability, but none of which are likely to admit a polynomial time approximation scheme. The family of problems we consider is the independent set problem on k–iterated total graphs for $k \geq 1$.

The other work along these lines concerns the approximability of domi-

nation problems which are among the most well-studied and useful graph–theoretic problems. Domination problem has several variants; while domination, connected domination and total domination can be approximated to a factor of $O(\log N)$, where N is the number of vertices, the other variants like independent and clique domination appear to be much harder to approximate. We first prove that independent domination cannot be approximated to a constant factor unless **P**=**NP** using a property called "self–improvement" which has found use for other problems like CLIQUE as well. Then, using problem–specific techniques, we prove that approximating independent and clique dominating sets to a factor of $N^{1-\epsilon}$ is **NP**-hard for any $\epsilon > 0$. While this extreme behavior in terms of inapproximability is also shared by the CLIQUE problem (due to a recent result by Hastad), we prove a still stronger negative result which states that both these problems cannot be approximated even to a factor of $N/\log^{1+\epsilon} N$, for any $\epsilon > 0$, unless the **NP**-complete problem 3SAT can be decided in sub–exponential deterministic time (which is considered unlikely). This is the **strongest negative result** we are aware of regarding the approximability of an unweighted **NP**-complete graph optimization problem.