6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 7: Reducibility, Rice's Theorem

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Problem 1: These are the key concepts from lecture this week:

1. Mapping Reducibility - pages 189-194 (make sure you understand Theorems 5.16, 5.17, 5.22, and 5.23)
2. Rice's Theorem

Problem 2: Give yourself the following test, then check your answers on the back of the handout. Classify each of the following problems as either

- **(D)** decidable,
- **(R)** recognizable but not decidable,
- **(C)** co-recognizable but not decidable, or
- **(N)** neither recognizable nor co-recognizable,

and indicate which undecidable examples follow from **Rice's Theorem**.

1. EQ_{NFA}, the Equivalence problem for NFA's.
2. $\{\langle M\rangle |\ M$ is a Turing Machine that runs for at least n steps when started with a blank input tape, where n is the length of the string $\langle M\rangle\}$.
3. $\{\langle M\rangle |\ M$ is a Turing Machine that accepts at least two inputs$\}$.
4. EQ_{TM}

Problem 3: (Mapping Reducibility) Answer the following True or False:

6. A_{TM} is mapping reducible to E_{TM}.
7. $A_{TM} \leq_m 0^*1^*$.

Problem 4:(Applications of Rice's Theorem and Mapping Reducibility)

1. Let $L_1 = \{< M > |M$ accepts 01 in a perfect number of steps $\}$. Show that L is undecidable. Does Rice's Theorem apply?

   Answer: Rice doesn't apply. Show that $A01 \leq_m L$.

2. Let $L_2 = \{< M > |L(M)$ is recognized by a TM having an even number of states. $\}$. Show that L_2 is decidable.

   Answer: Even though we have a language property, notice that any language has the property, so Rice's Thm doesn't apply.

3. Let $L_3 = \{< M > | L(M)$ is not regular $\}$. Show that L_3 is undecidable.

   Answer: Rice's Thm applies.

Problem 5: (Rice's Theorem and Mapping Reducibility)
Consider the problem of testing whether a Turing machine M accepts any binary string with an odd number of zeros.

1. Formulate this problem as a language; call it $ODDZ$.
2. Show that $ODDZ$ is undecidable.

   Answer: Use Rice's Theorem, show hypotheses are satisfied.
3. Is $ODDZ$ Turing-recognizable? Prove your answer.

   Answer: Yes, by running the TM in parallel (i.e., using the dove-tailing technique from class) on all inputs strings with an odd number of zeros until it accepts.
4. Is $ODDZ$ co-Turing-recognizable? Prove your answer.

   Answer: no, undecidable, but recognizable

Problem 2 Solutions:

1. D; recall the EQ_{DFA} algorithm from textbook.
2. D; just simulate M for up to $|\langle M \rangle|$ steps.
3. R; Undecidable by Rice's Theorem; Recognizable by running the TM in parallel (i.e., using the dove-tailing technique from class) on all input strings until it accepts two strings.
4. N; Undecidable by Rice's Theorem; Neither recognizable nor co-recognizable from textbook.

Problem 3 Solutions:

1. False; A_{TM} is recognizable, E_{TM} is not. See Corollary 5.17.
2. False; 0^*1^* is decidable, A_{TM} is not. See Theorem 5.16.