6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch

Homework 11

Due: April 30, 2007 *Elena Grigorescu*

Readings: Sections 7.4, 7.5, Garey-Johnson (optional)

Problem 1:
(Sipser 7.39) In the proof of the Cook-Levin theorem (for NP-completeness of SAT), we defined a tile to be a 2 by 3 rectangle of cells. Show why the proof would have failed if we had used 2 by 2 windows instead.

Problem 2:
(Sipser 7.36) Show that, if P=NP, a polynomial time algorithm exists that, given a Boolean formula ϕ, actually produces a satisfying assignment for ϕ if it is satisfiable.
(Note: NP is a class of *languages* and above you are being asked for an *algorithm* that produces a satisfying assignment (if one exists) for a given ϕ. Thus simply saying that, "because SAT is in NP, you are done" isn't enough.)

Problem 3: (Sipser 7.20) Let G represent an undirected graph and let

$$SPATH = \{\langle G, a, b, k\rangle |\ G \text{ contains a simple path of length at } \textbf{most } k \text{ from } a \text{ to } b\}$$

and

$$LPATH = \{\langle G, a, b, k\rangle |\ G \text{ contains a simple path of length at } \textbf{least } k \text{ from } a \text{ to } b\}$$

1. Show that SPATH $\in$ P.
2. Show that LPATH is NP-complete. You may assume the NP-completeness of UHAMPATH, the Hamiltonian path problem for undirected graphs.

Problem 4: (Sipser 7.23) A **cut** in an undirected graph is a separation of the vertices V into two disjoint subsets S and T. The size of a cut is the number of edges that have one endpoint in S and the other in T. Let

$$MAXCUT = \{\langle G, k\rangle |\ G \text{ has a cut of size } k \text{ or more}\}.$$

Show that MAXCUT is NP-complete. You may assume the result of Problem 7.24 of Sipser's book.

(Hint: Show that $\neq$-SAT $\leq_P$ MAXCUT. The variable gadget for variable x is a collection of $3k$ nodes labeled with x and another $3k$ nodes labeled with $\overline{x}$, where k is the number of clauses. All nodes labeled x are connected with all nodes labeled $\overline{x}$. The clause gadget is a triangle of three edges connecting three nodes labeled with the literals appearing in the clique. Do not use the same node in more than one clause gadget. Prove that this reduction works.)