Quiz 2 Practice Problems

1 Sorting

1. Fill in either True or False for whether each sorting algorithm is in-place and stable. Also fill in the running time in terms of the number of elements n and the range of the elements k.

	in-place	stable	running time
Insertion Sort	yes	yes	$O(n^2)$
Counting Sort	no	yes	$O(n+k)$
Selection Sort	yes	no	$O(n^2)$
Heap Sort	yes	no	$O(n \log n)$
Merge Sort	no	yes	$O(n \log n)$
Radix Sort	no	yes	$O((n+2^r)(\log k)/r)$ or $O(n \log k / \log n)$

2. Given a list of n positive integers all less than $k = n^2$, would you rather use Counting Sort or Selection Sort? Why?

   Solution: It depends. Both have the same asymptotic running time. If memory is limited, one might want to use Selection Sort, because it is in-place; if preserving the order of integers with the same value is important, one might use Counting Sort, because it is stable.

2 Heaps

1. Show that, with the array representation for storing an n-element heap, the leaves are the nodes indexed by $\lfloor \frac{n}{2} \rfloor + 1, \ldots, n$.

   Solution: These nodes are precisely the nodes with no children, since $2i$ and $2i+1$, the indices where their children would lie, are past the end of the heap. One can also verify that all nodes earlier in the array do have children in the heap.

2. Is the sequence $< 21, 15, 18, 8, 12, 11, 16, 4, 9 >$ a max-heap? Justify.

   Solution: No. Node 4 (a key of 8) is smaller than its right child, node 9 (a key of 9).

3 DFS

Prove or disprove: Given two vertices u and v with discovery times $d[u] > d[v]$, u must be a descendant of v in G.
Solution: Disproof: u and v can be in different branches of the DFS tree, where v's branch is visited first.

4 Shortest Paths

You have an undirected weighted graph G, a source s, shortest path estimates $d[u] = 50$ and $d[v] = 40$, and an edge with weight $w(u, v) = 5$.

1. What happens when you call `Relax(u,v)`?

   Solution: Nothing. v's distance estimate is not improved.

2. What happens when you call `Relax(v,u)`?

   Solution: u's distance estimate is improved to $d[u] = 45$.

3. If you are told that the shortest path weight $\delta(s, u) = 45$, what can you say about the shortest path weight $\delta(v, u)$? Why?

   Solution: $\delta(v, u) = 5$. We have already found a path from s to v to u of weight 45. Since we are told this is also the shortest path weight, our path from s to v to u must be a shortest path. We know that subpaths of shortest paths are shortest paths, so our path from v to u of weight 5 must also be a shortest path.