ITCS 2016

Timeability of Extensive-Form Games

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joint work with
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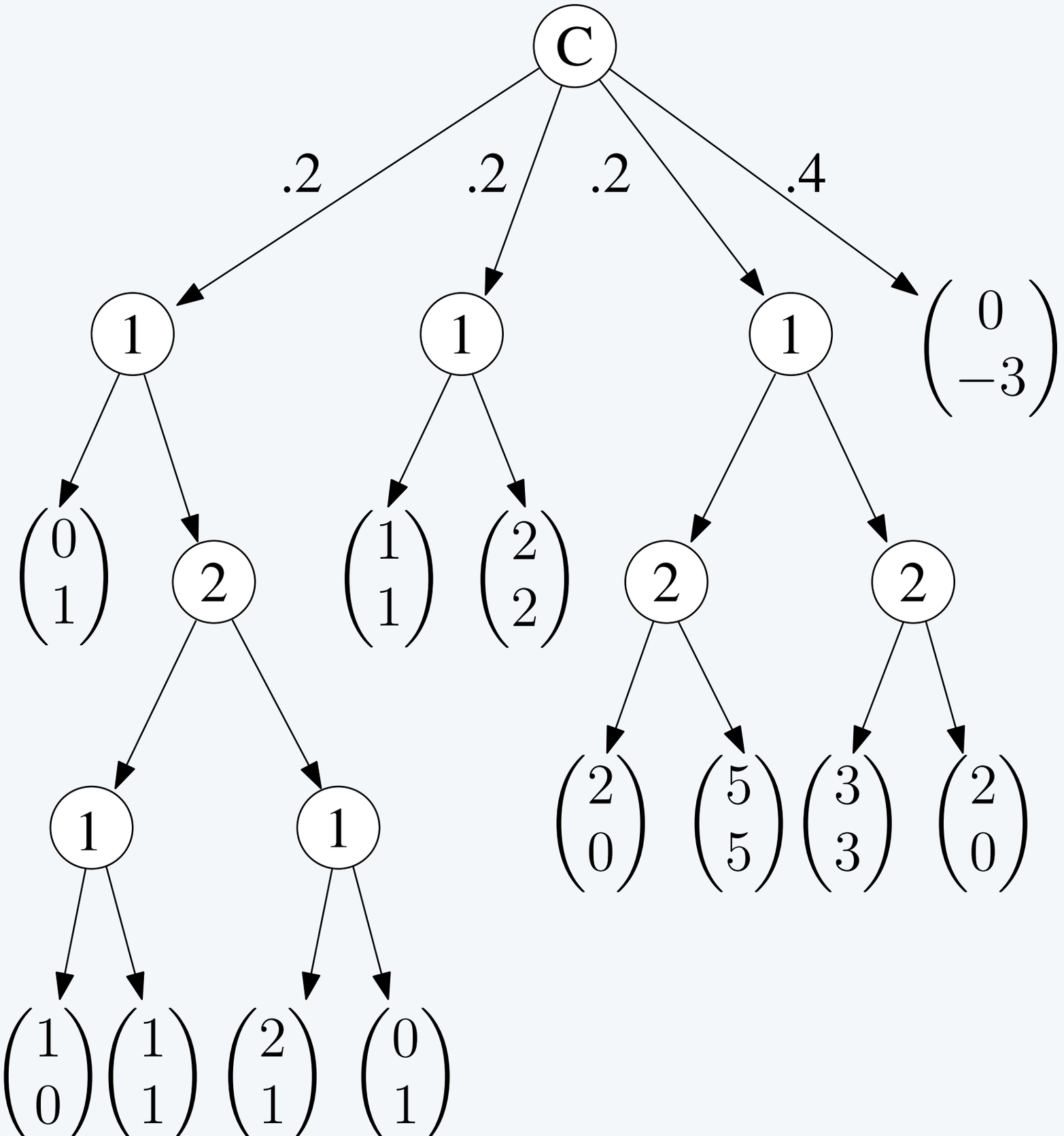
Extensive-form games



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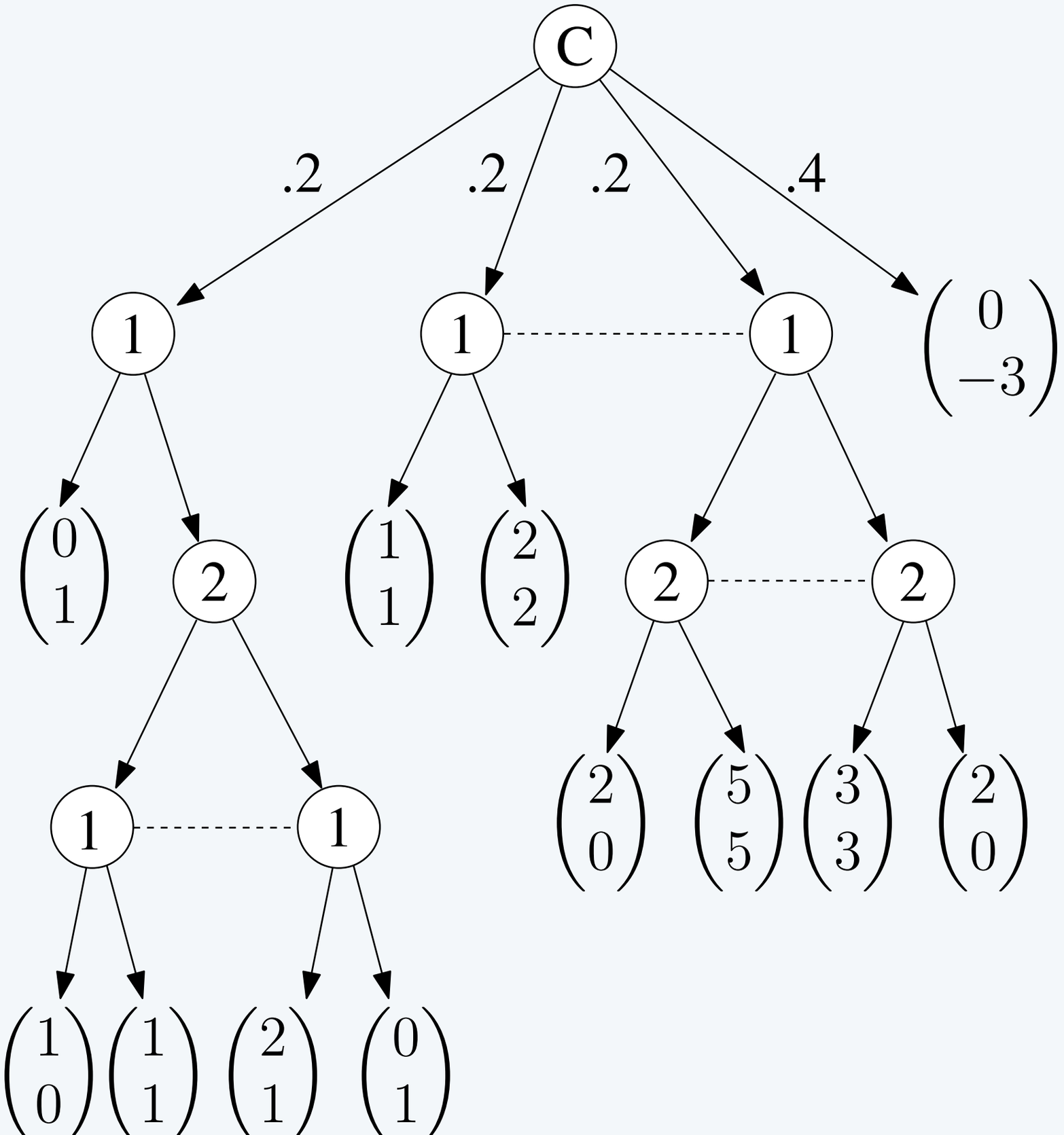

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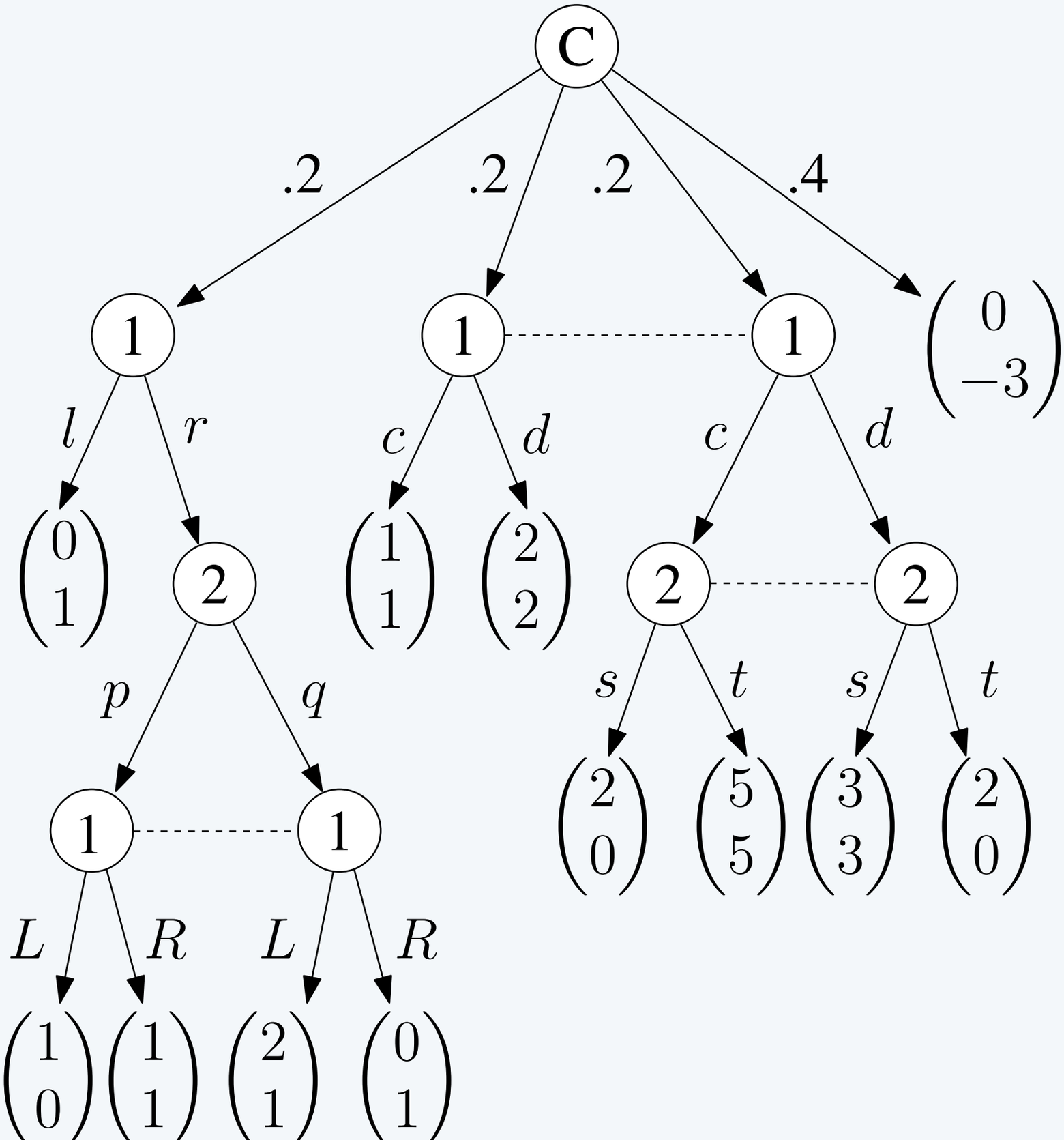

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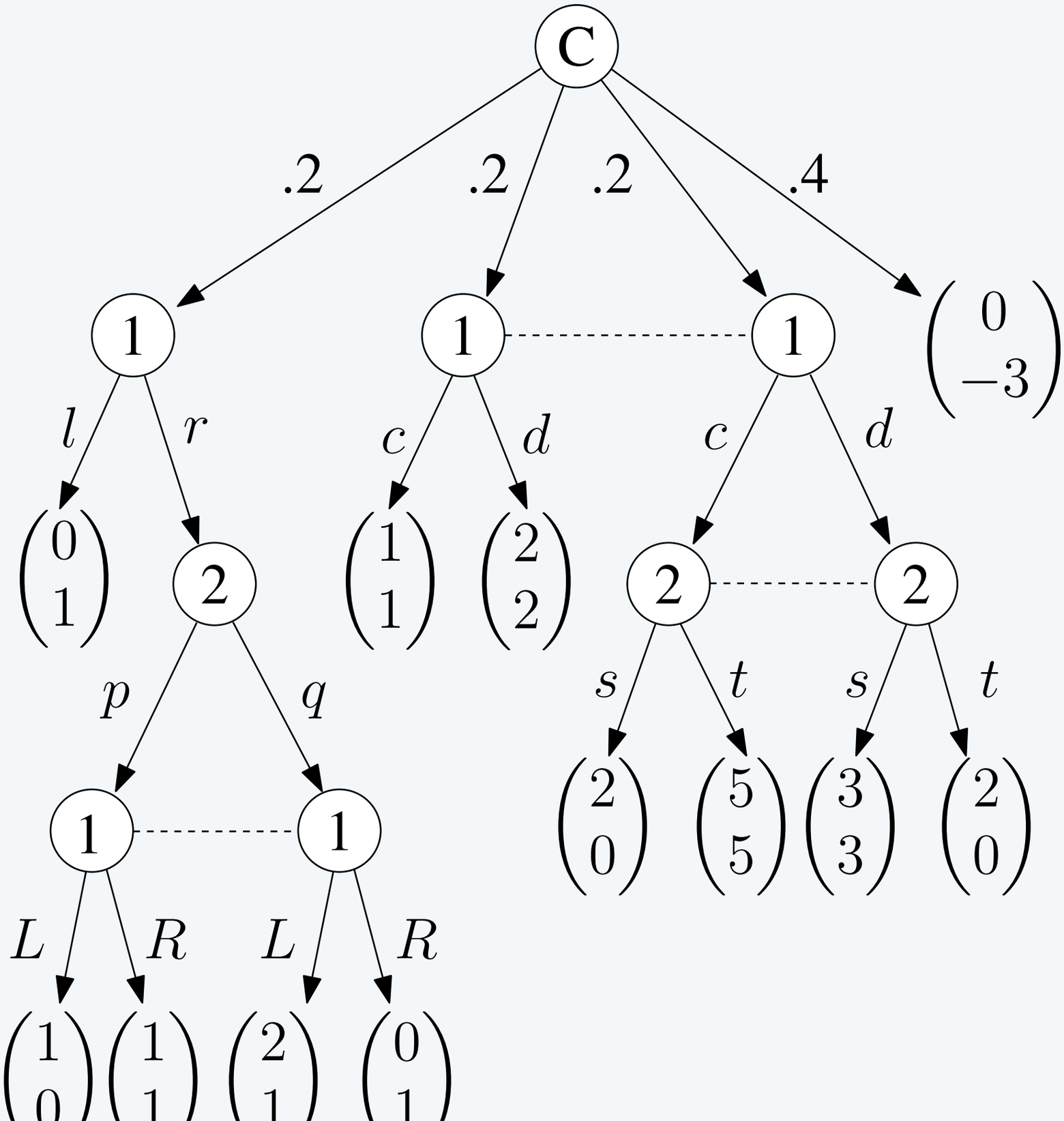

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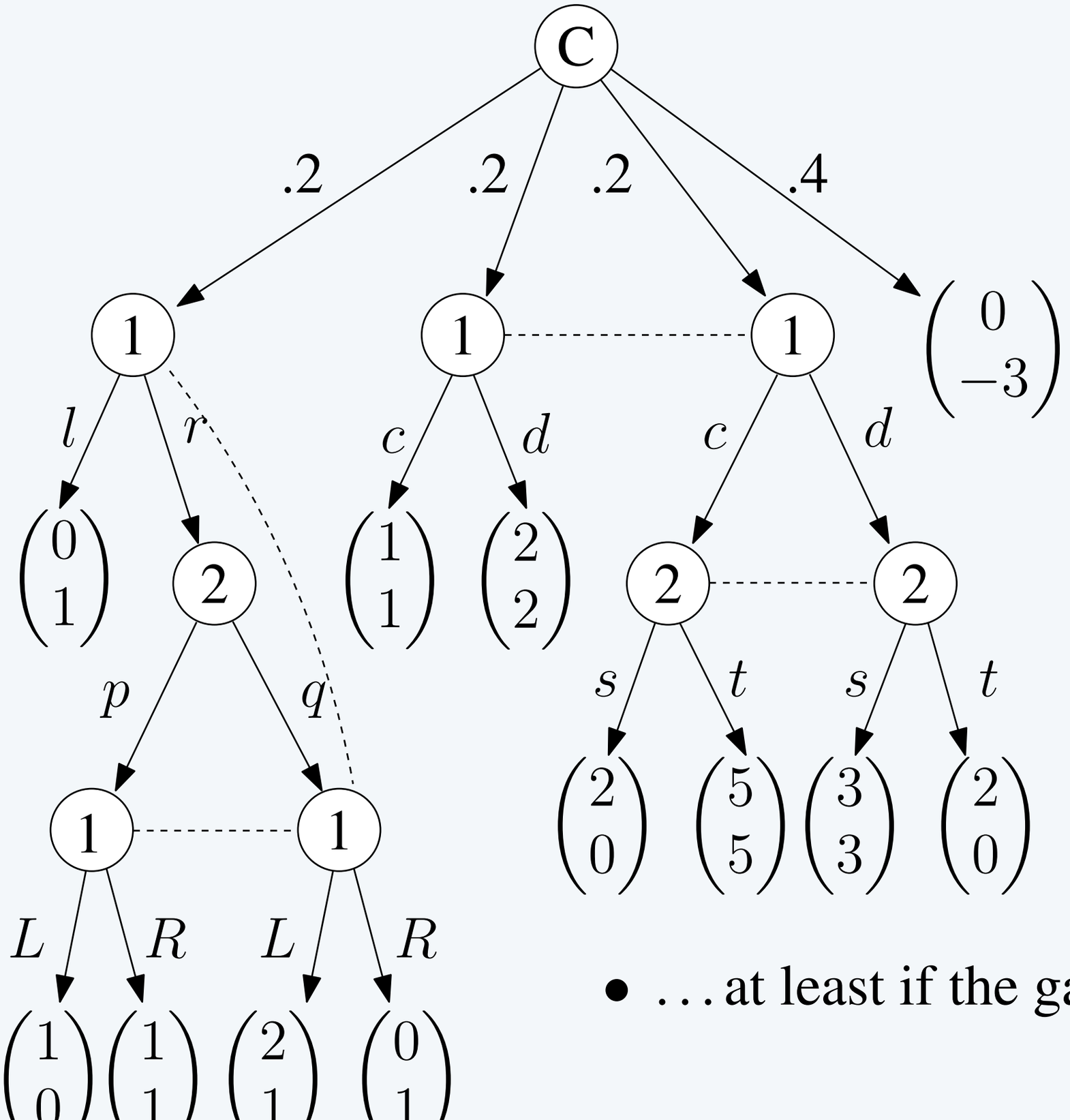
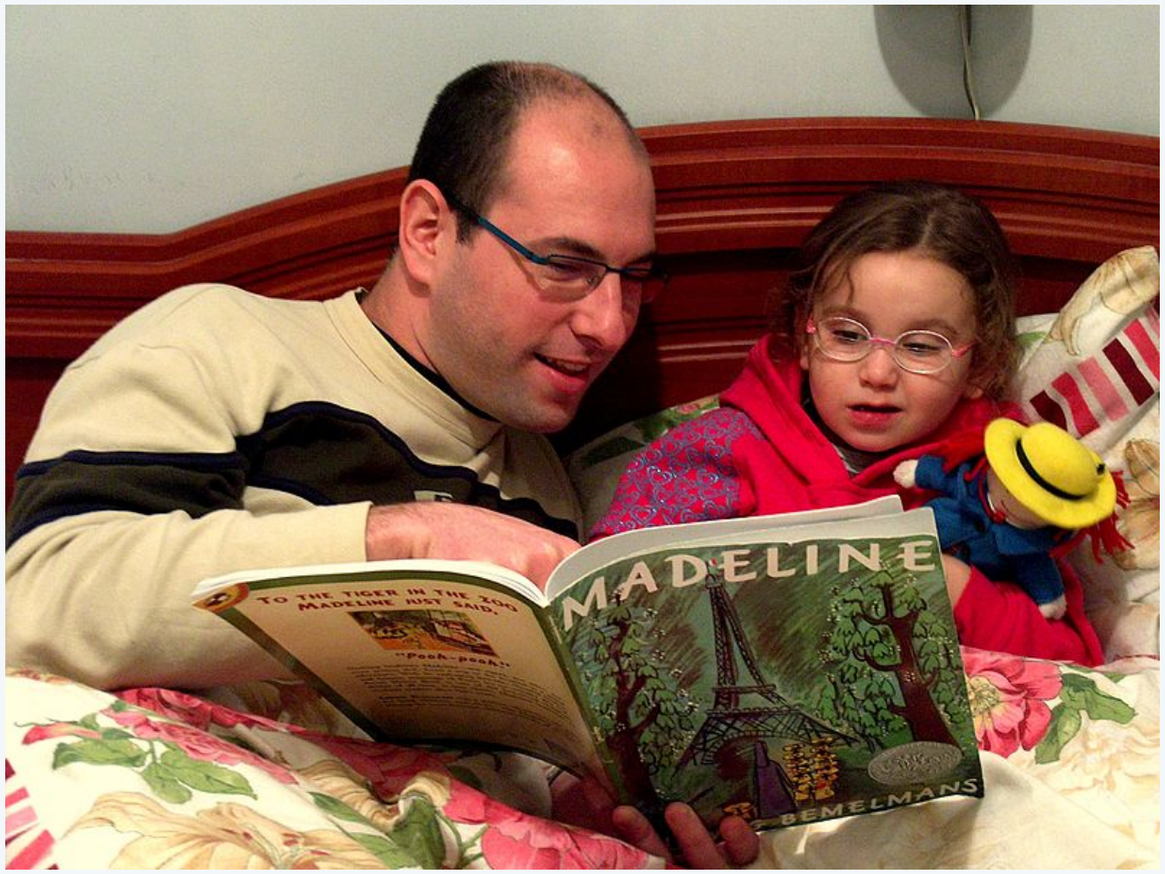


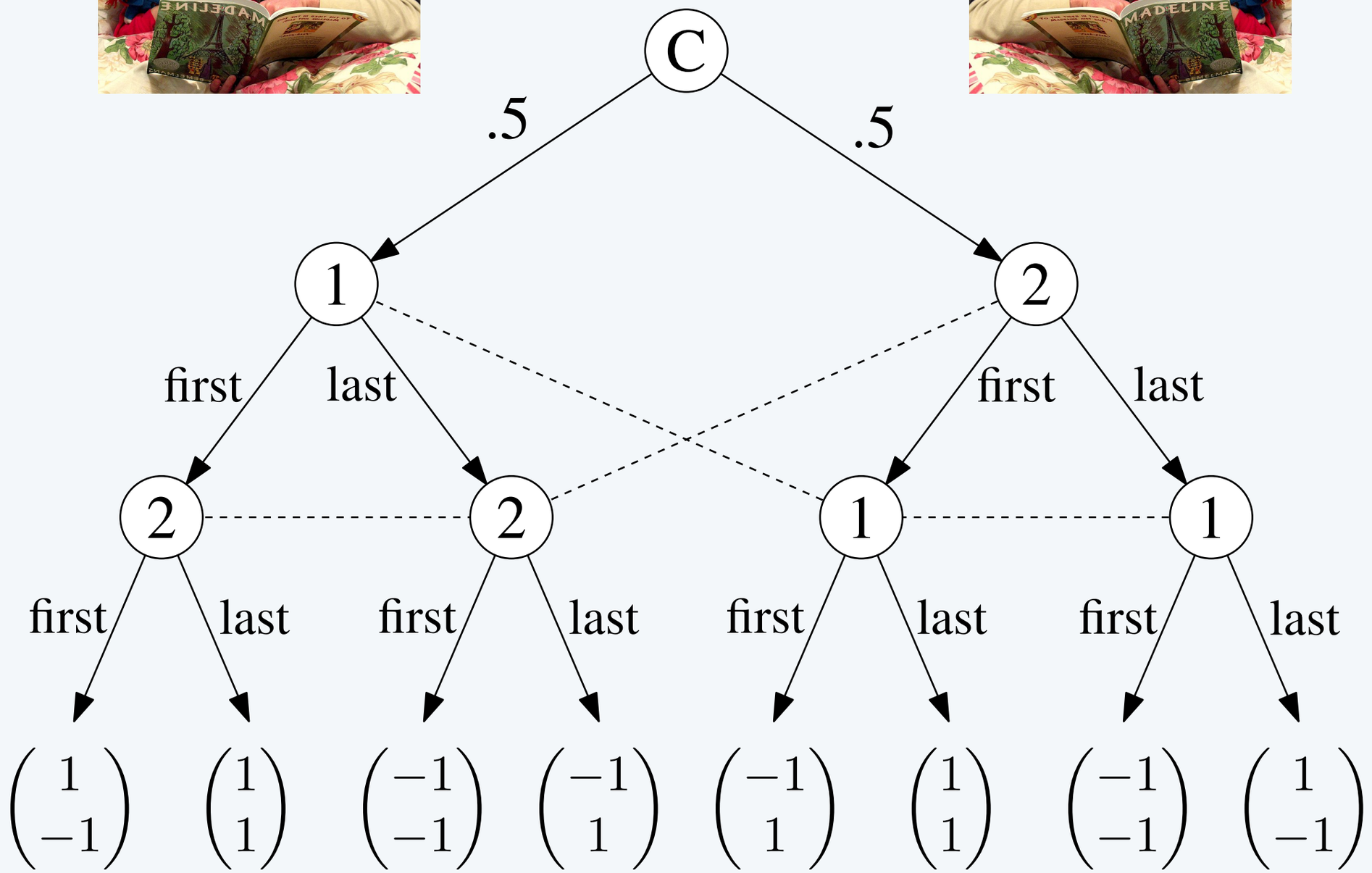
- Built on a game tree.
- Information sets model nodes that a player cannot distinguish.
- Labels map actions to outgoing edges.
- Close correspondence between model and real world.
- . . . at least if the game has *perfect recall*.

Example: bedtime

- It is bedtime for your two kids.
- They are very competitive about who gets tucked in first.
- As a good game theoretician, you secretly flip a coin to decide who to read to first.



As an extensive-form game



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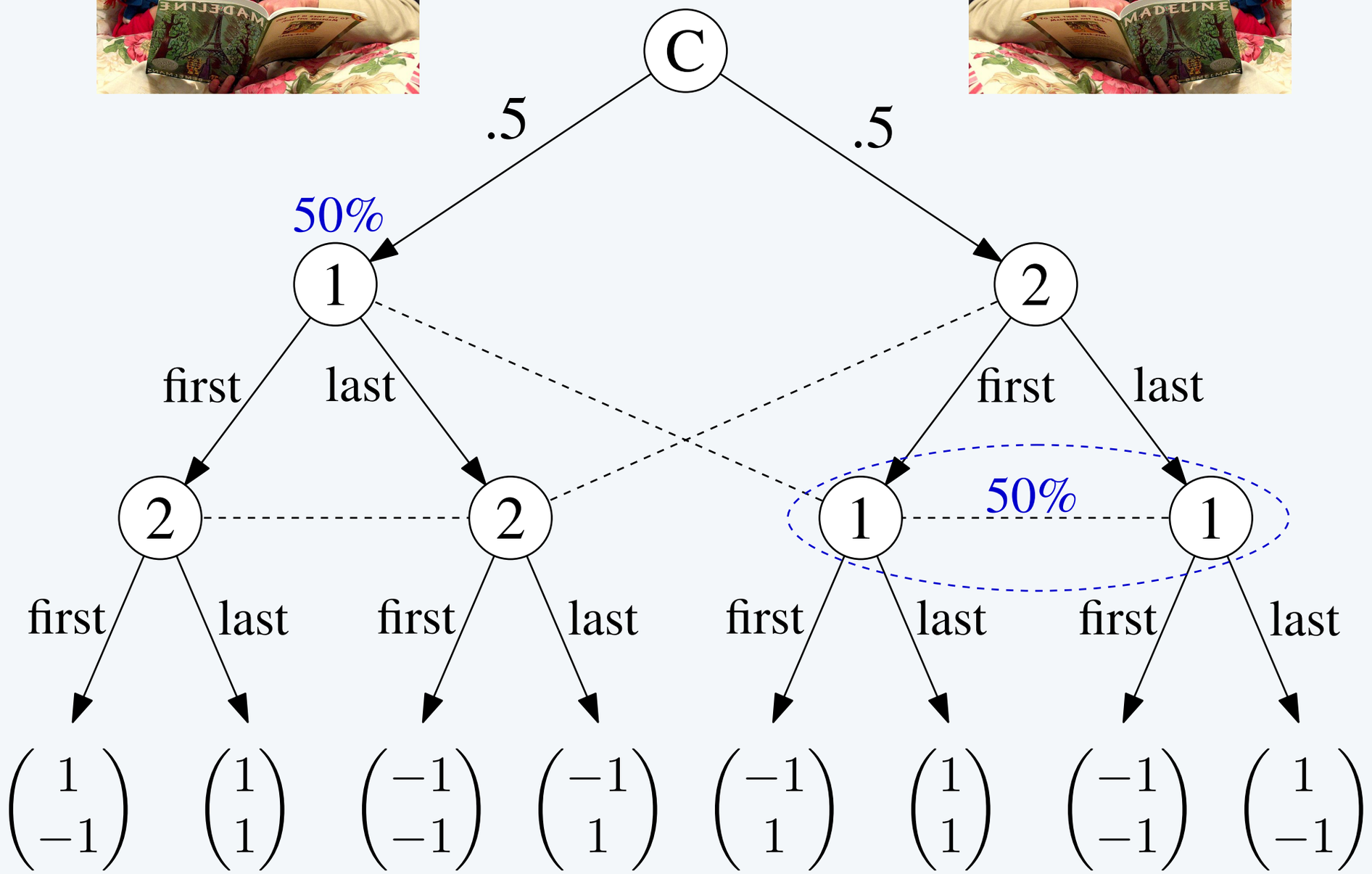

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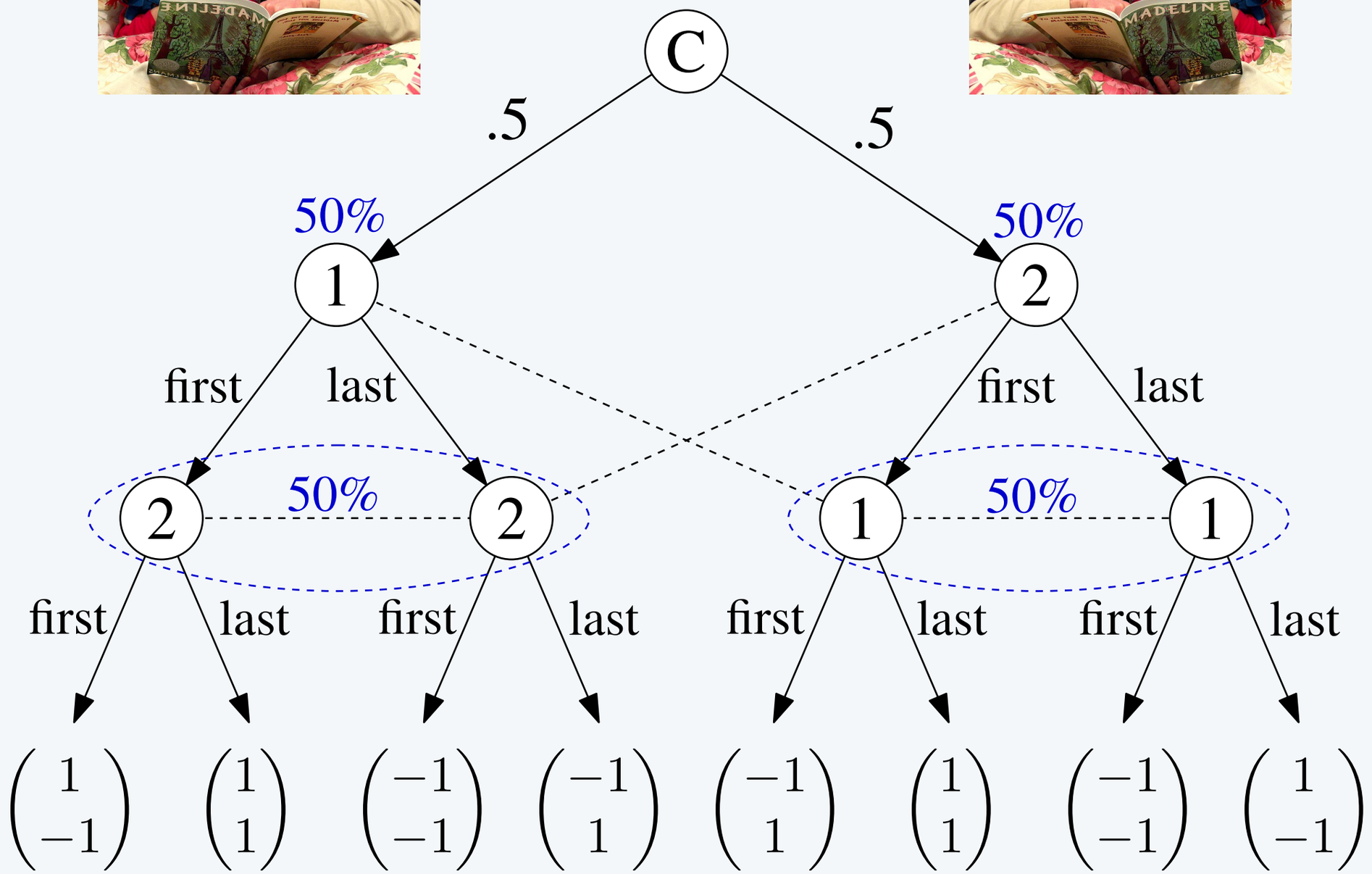

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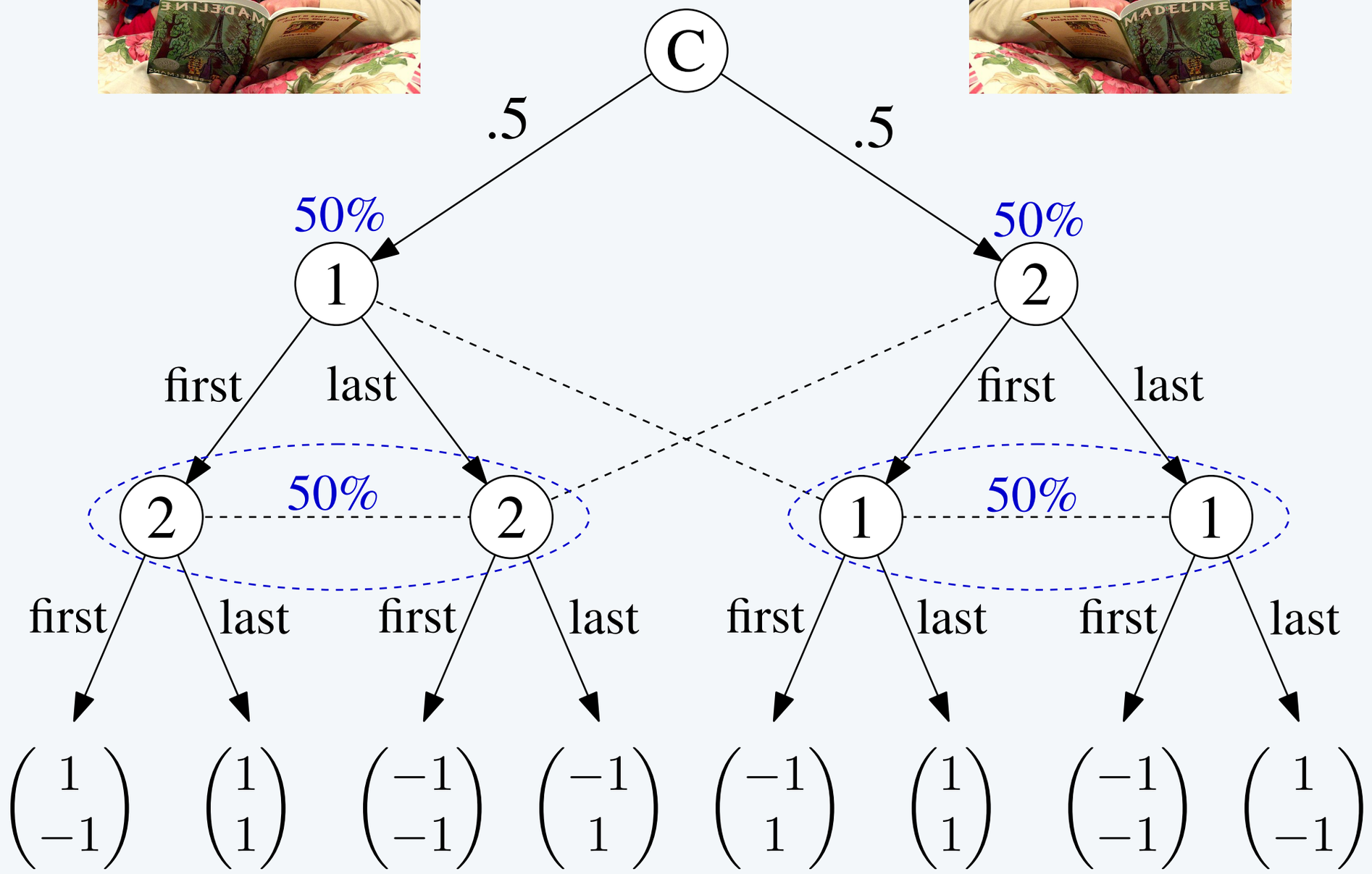

- All strategies yield expected utility 0.

Time will tell

- We have forgotten to model time!

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- Agents can observe time.

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- Every action taken takes time.

Time will tell

- We have forgotten to model time!

- Agents can observe time.

- Every action taken takes time.

- Sometimes, time itself reveals information.

With time included

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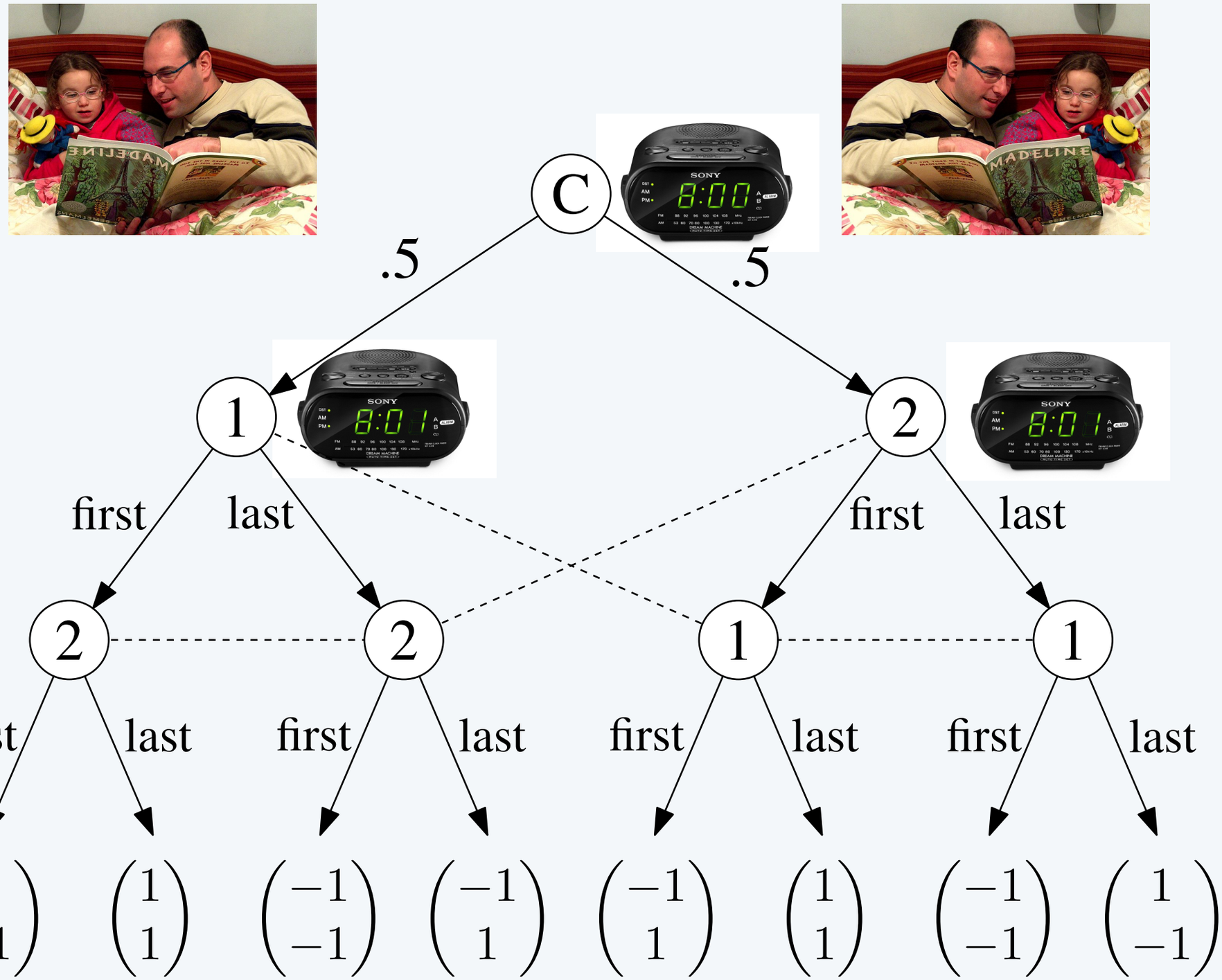

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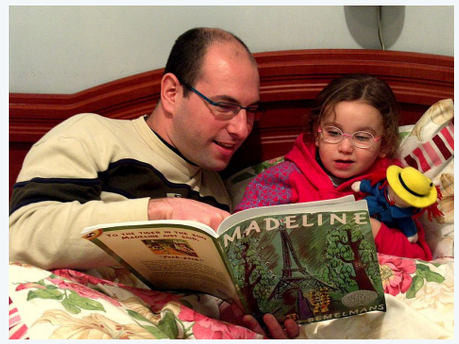

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With time included

- No matter what we do, someone will learn something they weren't supposed to.

Timing a game

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- When it is a player's turn to move, she also learns the time.
- A randomized timing is a probability distribution over deterministic timings, sampled before the game starts.
- Example: Getting the kids to sleep:
  - Wait a random amount of time before tucking in the first kid
  - If we wait uniformly between 1 and n minutes, then correct beliefs are preserved for all but 2 times, i.e., with prob. $\frac{n-2}{n}$.

ϵ-timeable and exactly timeable

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Theorem The following are equivalent:

- The game is deterministically timeable
- The game is randomized 0-timeable
- The game can be redrawn with each information set "levelled"
- There is a total order on the information sets that respects the order from the game tree

Bounds on timeability

Definition Let $\exp_2^n(x) = 2^{2^{\cdots^{2^x}}}$ where the tower contains n 2's.

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- Above holds true, even if we are allowed access to relativistic time dilation, as long as not by more than a constant factor.

Example: An onion routing protocol

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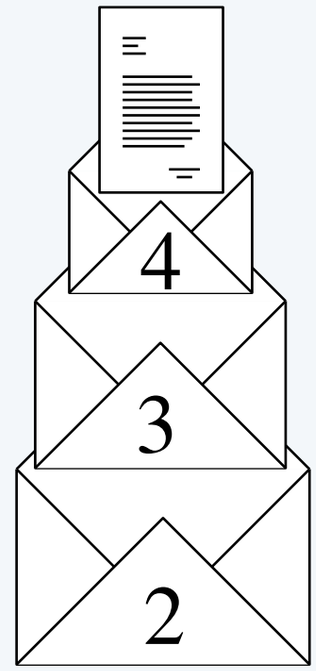
  - Send nested enveloped to intermediary players
  - Intermediaries can't tell original sender or receiver



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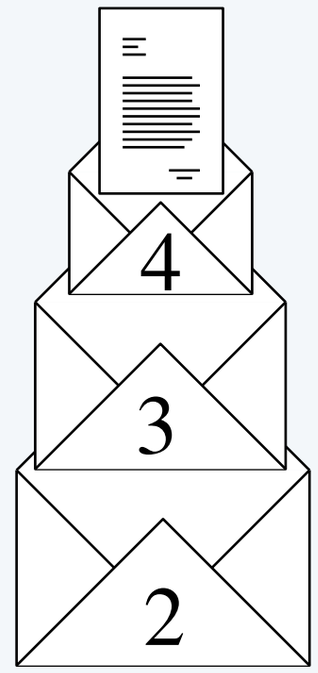
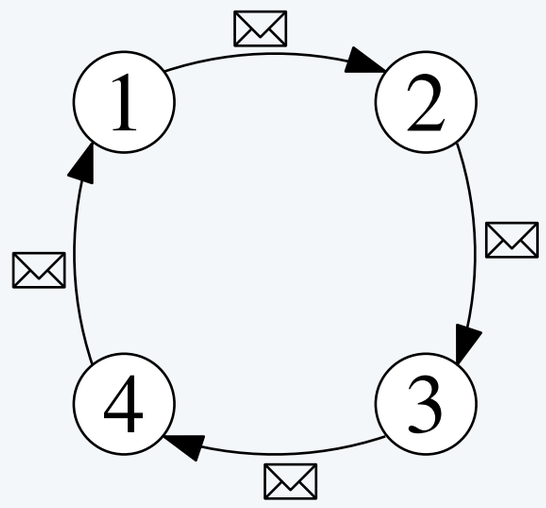


- A "careless" network design, that works when ignoring time:



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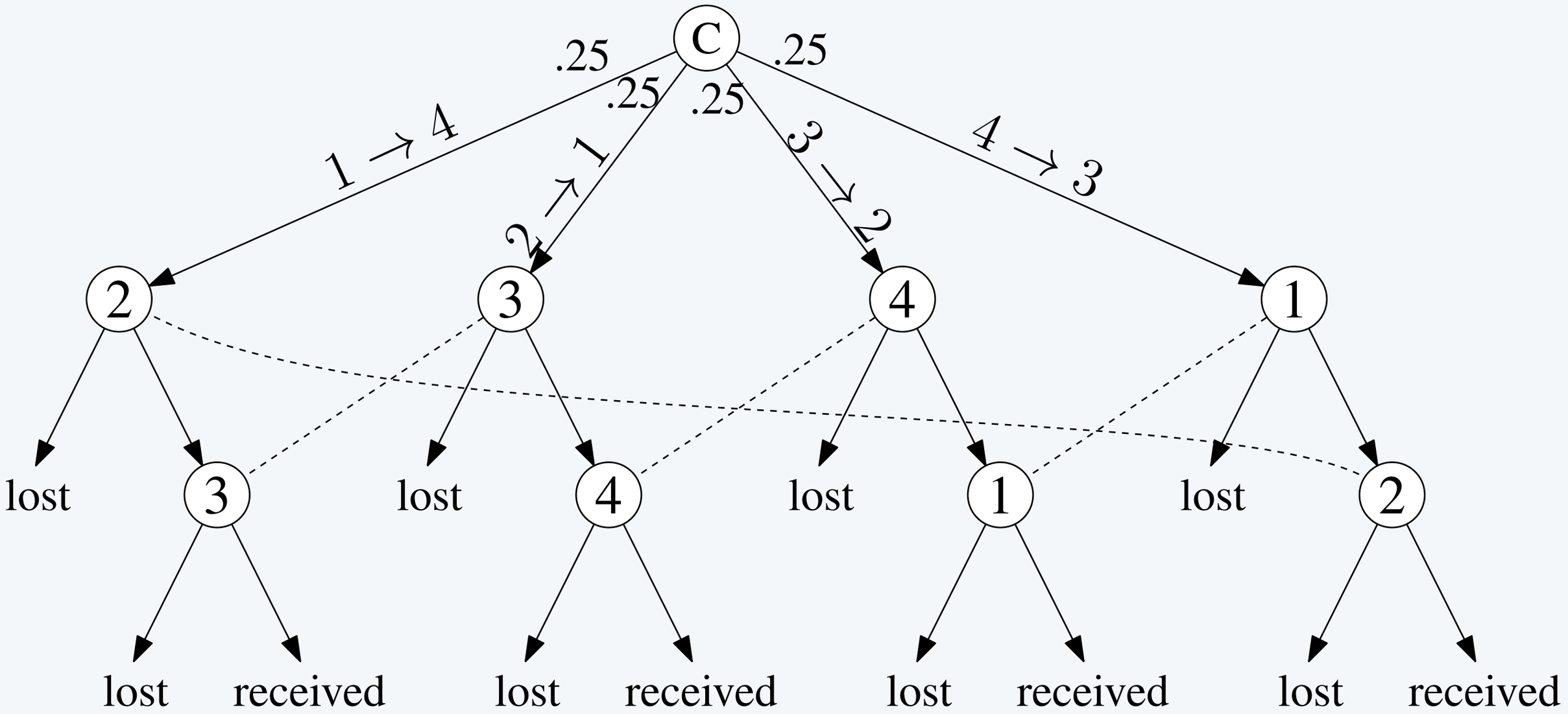

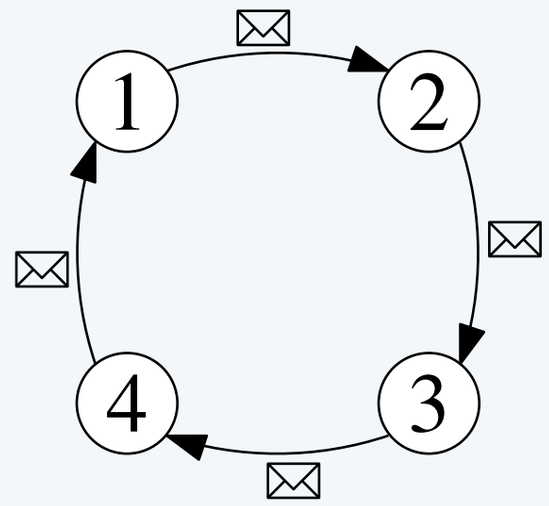

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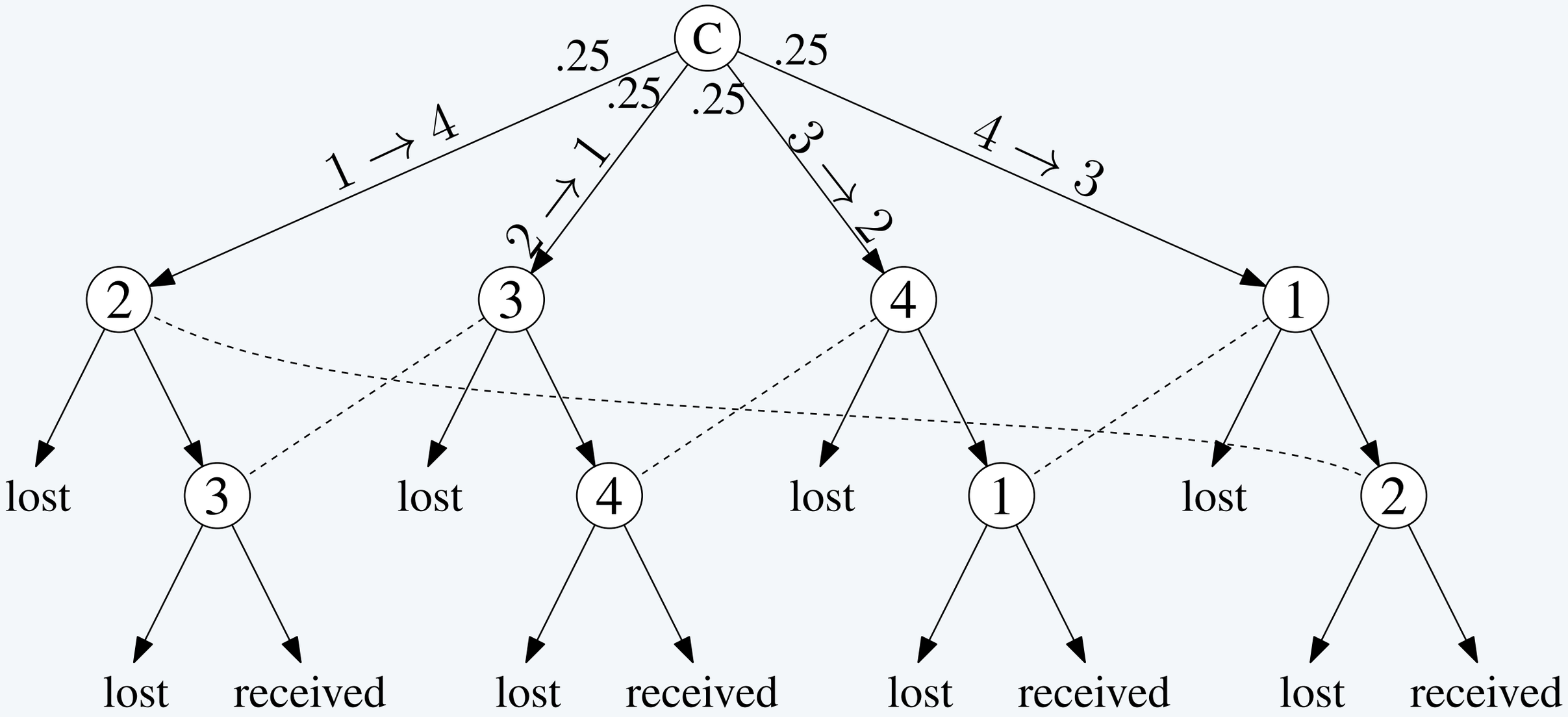
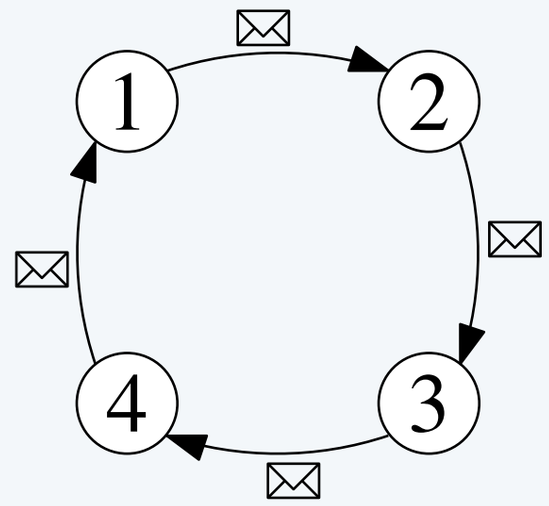


- Player i's utility:
  - If message passes from $i-1$ to $i+2$: $1+\epsilon$
  - If message passes from $i-2$ to $i+1$: -1



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- Kroer and Sandholm (2014) implicitly assumes that all extensive-form games are timeable.

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Future research:

- What do we gain by assuming timeability?
- Given a game and ϵ, how much time is needed to ϵ-time the game?
- How do approximate timings affect equilibria?